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Synchronization Effects in the Interaction of Complex Nonlinear Systems

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ABSTRACT

This study investigates the fundamental mechanisms of synchronization in nonlinear dynamical systems under external influences, bridging theoretical analysis with real-world geophysical applications. We first focus on the van der Pol nonlinear oscillator to explore forced synchronization, employing numerical modeling to analyze how external periodic forces modify the system's phase and frequency dynamics. A key finding reveals that the introduction of random noise can, counter-intuitively, contribute to the regulation of complex or chaotic regimes, leading to enhanced stability. Extending this analysis to the Sun-Earth connection, we utilize the wavelet coherence method to quantitatively assess the time-frequency coherence between solar parameters (such as the f10.7 index and the IMF B_z component) and terrestrial responses. This analysis successfully detects significant synchronization effects, confirming that the coherent interaction between these systems is directly responsible for triggering geomagnetic storms. Overall, this work provides a comprehensive framework for understanding synchronization phenomena, from controlled nonlinear systems to large-scale astrophysical interactions.

Key words: *Synchronization, Van der Pol Oscillator, Wavelet Coherence, Stochastic Resonance, Geomagnetic Storms.*

Introduction

Synchronization is a universal organizing principle describing the process by which two or more interacting dynamical systems operate in a time-coordinated manner. Using numerical modeling, we proceed to analyze how applied external periodic forces and the introduction of random noise collectively influence the oscillator’s characteristic phase and frequency behavior. Crucially, we seek to determine the precise conditions under which noise often seen as a disruptive element can surprisingly contribute to the regulation of chaotic regimes, thereby inducing stability through stochastic resonance [1-2]. Using numerical modeling, we analyze how external periodic forces and the introduction of random noise change the oscillator’s phase and frequency behavior. A key area of focus is determining the conditions under which noise can surprisingly contribute to the regulation of chaotic regimes, thereby inducing stability through stochastic resonance.

Beyond laboratory models, synchronization is fundamental to solar-terrestrial coupling the complex interaction chain through which solar energy influences the Earth’s Geospace environment. To evaluate this coupling, we employ wavelet coherence, which provides a time-frequency assessment of the interaction between solar drivers and terrestrial signals. By analyzing specific parameters such as the IMF (B_z) component and the solar flux index F10.7, this study identifies synchronization windows that trigger geomagnetic storms on Earth, offering a unified framework that links nonlinear oscillator dynamics with large-scale space-weather phenomena and clarifies how coherent external forcing shapes system-wide responses.

Materials and methods

This article briefly considers Causes of geomagnetic storms on Earth in 1960-2025, 27 days apart. Based on the available study, some characteristics of these cases are analyzed. The averaged data is taken from several stations on Earth.

The study is processed using proven methods of mathematical statistics, validated statistical and probability-based methods commonly used in magnetospheric physics and solar–terrestrial research.

Results

The van der Pol oscillator is a prototypical nonlinear self-sustained system characterized by a non-linear damping regime in the absence of external forcing.

$$\frac{d^2x}{dt^2} - (\varepsilon - x^2)\frac{dx}{dt} + \omega^2 x = 0 \quad (1)$$

To analyze the system's dynamics, the Fast Fourier Transform (FFT) is employed, which decomposes complex signals into simpler frequency components. The first case examined is the unforced oscillator. As shown in Figure 1, with no external influence, the oscillator oscillates at its natural frequency, which is clearly represented by the fundamental peak corresponding to the selected frequency data.

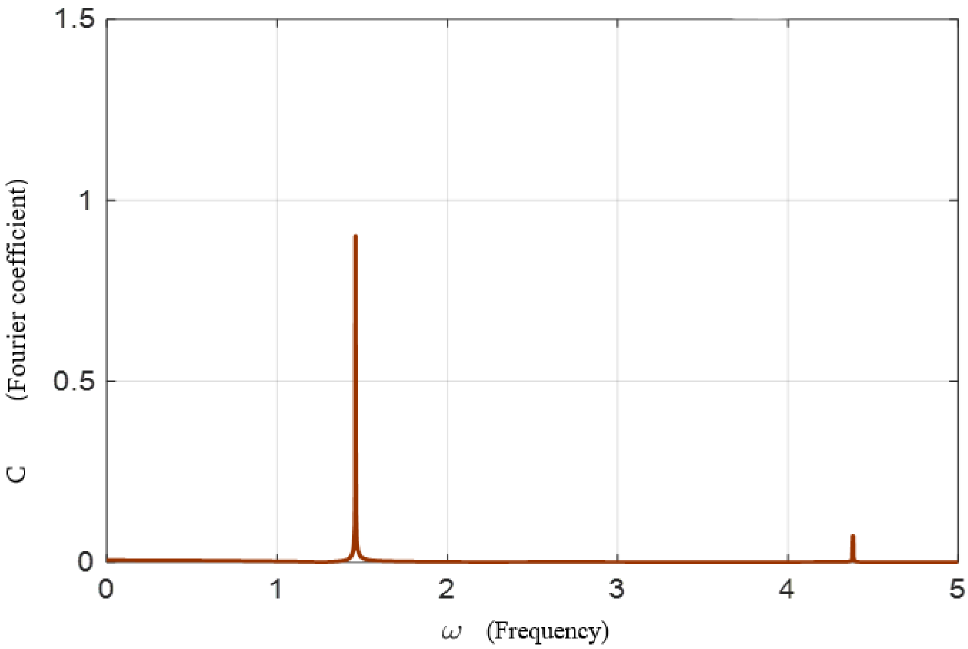

Fig.1. Oscillator without external force. ε =1;ω= 1.5;

When the oscillator is subjected to an external force, the governing equation takes the form:

$$\frac{d^2x}{dt^2} - (\varepsilon - x^2)\frac{dx}{dt} + \omega^2 x = A\sin(\Omega t) \quad (2)$$

In this case, the external force is a harmonic excitation. The application of this force yields two possible outcomes: synchronous and asynchronous regimes.

Asynchronous Regime (Figure 2): Figure 2 shows the frequency spectrum when the amplitude of the external force is relatively small, insufficient to force the system into synchronization with its frequency. The result is an asynchronous regime, clearly indicated by the distinct, separate frequency peaks of the external force and the oscillator's natural frequency. They do not coincide.

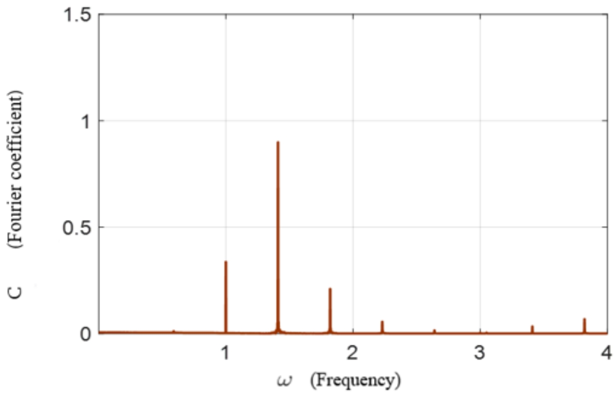

Fig.2. Asynchronous mode
$\varepsilon = 1;\ A = 1;\ \omega = 1.5;\ \Omega = 1;$

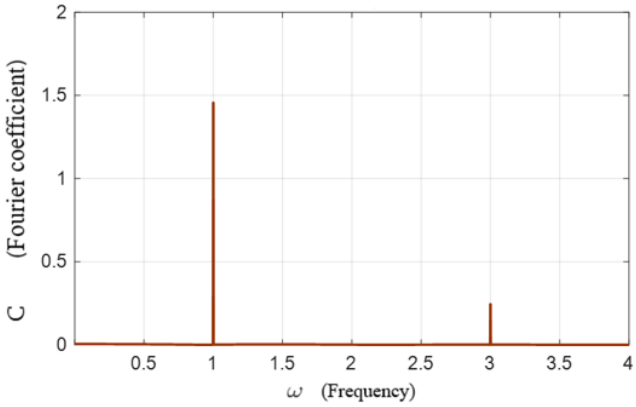

Fig.3. Synchronization without force.
$\varepsilon = 1; A = 4; \omega = 1.5; \Omega = 1;$

Synchronous Regime (Figure 3): To demonstrate synchronization between the external force and the oscillator, the amplitude of the external force was increased while keeping the frequency parameter constant. The increased amplitude successfully induced synchronization, which is evident in the spectrum by the presence of a single fundamental frequency component (Figure 3), which now matches the frequency of the external force.

We next observe the van der Pol oscillator when the system is subjected to a random, unpredictable force, or noise. The equation is modified to include a noise term:

$$\frac{d^2x}{dt^2} - (\varepsilon - x^2)\frac{dx}{dt} + \omega^2 x = A\sin(\Omega t) + \sqrt{2b}B(t) \tag{3}$$

As shown previously, a small amplitude external force alone did not establish synchronization between the oscillator's natural frequency and the forcing frequency. However, the addition of white noise to the system, even with the small amplitude external force, successfully established the synchronization effect (Figure 4). This illustrates a key principle related to Stochastic Resonance, where noise aids coherence.

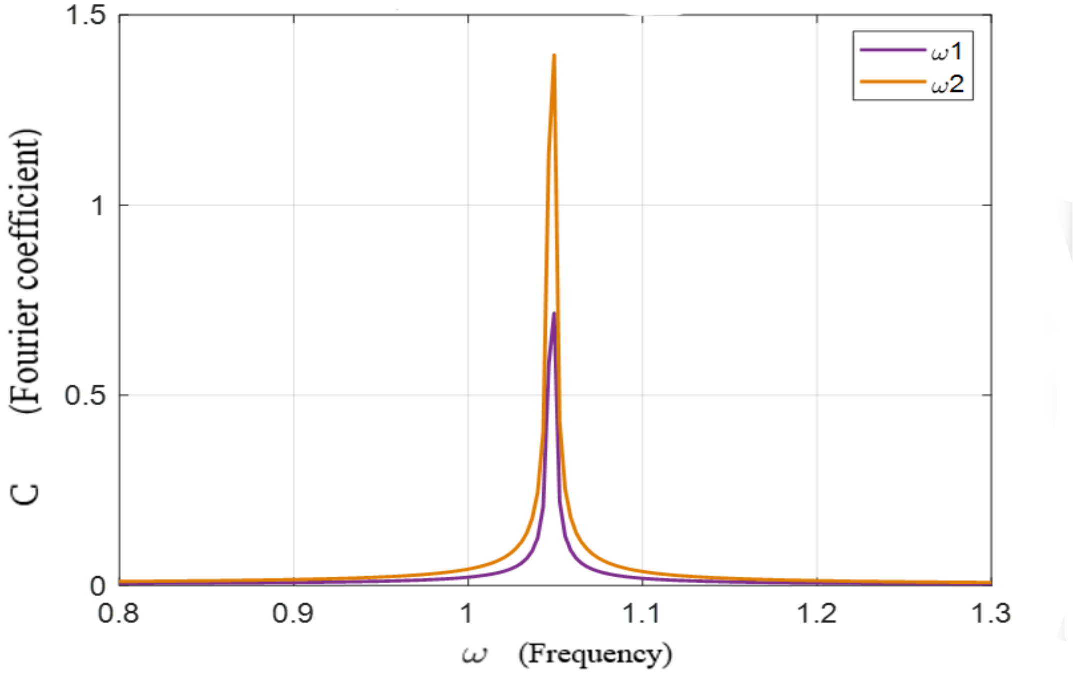

Fig.4. Synchronization of Van der Pol oscillators under the influence of white noise. ε=0,5; b=0.2;
Ω=1.5; ω=1; B=2.5;

Now let's look at the synchronization effect using the example of Adler's equation [3-4]. If the parameters included in the Van der Pol equation satisfy the given conditions: $\omega \approx \Omega$, $\varepsilon > 0, A \ll 1$, We look for the solution in the following form: $x(t) = R(t)cos(\omega t + \varphi(t))$, where A and φ are slowly varying functions of time, and A_0 and ω_0 are the amplitude and frequency of the self-oscillation *without* the external force. By substituting this solution and performing averaging (or other asymptotic methods), we obtain an equation for the phase (φ), which is called Adler's equation:

$$\dot{\varphi} = -\Delta + \frac{\beta}{R_{b\vartheta}} sin(\varphi(t)) \tag{4}$$

Synchronization can be classified as having either a continuous (constant) or intermittent (variable) character. We examined the phase synchronization behavior in the case of a single external force.

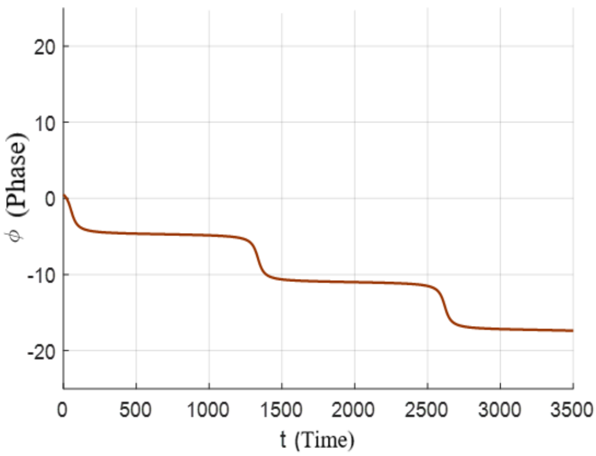

Fig.5.a. Phase synchronization for the Adler equation. Phase transitions. $\varepsilon = 0.1$; Δ=0.032; β =0.02;

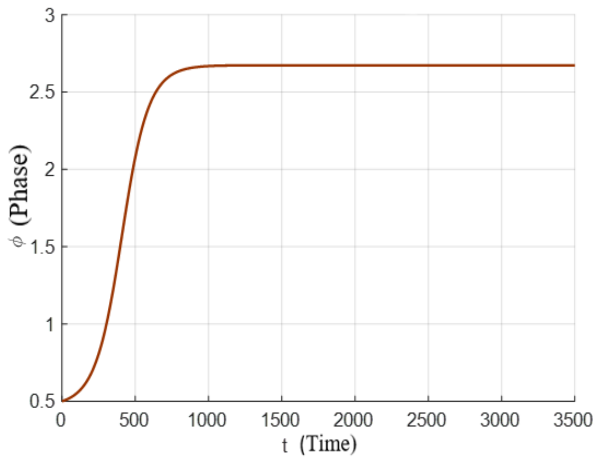

Fig. 5.b. Phase synchronization for the Adler equation. $\varepsilon = 0.1$; Δ=0.005; $\beta = 0.007$;

Intermittent Phase Synchronization (Figure 5a): Over a specific time interval, the phase remains constant, followed by a phase slip (a jump). The system then re-establishes a synchronization regime at a different phase value, and this cycle repeats (Figure 5a).

Continuous Phase Synchronization (Figure 5b): We can also observe a case where phase synchronization adopts a constant character. With the given parameters, synchronization did not occur over an initial time interval, but then took on a continuous (constant) character (Figure 5b).

In order to clarify how solar variability imprints itself on Earth's magnetosphere, we investigated long-term Sun–terrestrial coupling using multi-decadal records of the Dst geomagnetic index, the solar flux index F10.7, and the sunspot number R. The purpose of this analysis was to determine whether coherent structures, synchronization regimes, or phase-locking patterns could explain the emergence and statistical distribution of geomagnetic disturbances, including storm-time events. Our results show that synchronization between solar drivers and the geomagnetic response appears at several characteristic scales.

Wavelet coherence reveals a broad, persistent synchronization band near the 11-year solar cycle, accompanied by intermittent coherence patches around the 27-day rotational period, indicating that both long-term and recurrent solar features leave detectable signatures in Dst variability. These findings are supported by the F10.7-Dst scatter distribution, which shows weak but systematic coupling during quiet and moderate conditions, and by the R-Dst density plot, where most values cluster between 20 and 60 nT, reflecting the background state of the magnetosphere. Only a small fraction of points extend below $Dst <$ 100 nT, corresponding to intense geomagnetic storms.

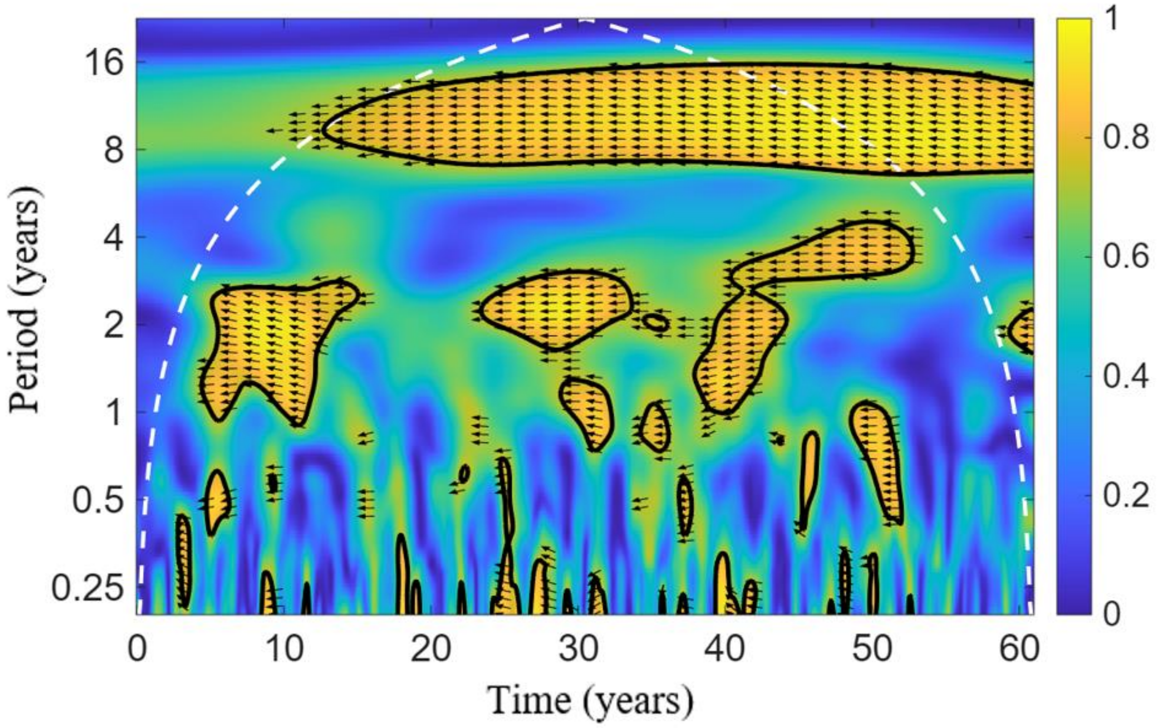

Fig.6. Coherence of the interplanetary magnetic field and the Earth's magnetic field

This figure.6 presents the wavelet coherence between the interplanetary magnetic field (IMF) and the Earth’s magnetic field. The coherence map displays scale-dependent correlations, where high-coherence regions highlight intervals of strong coupling between IMF fluctuations and geomagnetic responses. The cone of influence and significance contours outline the statistically reliable coherence zones.

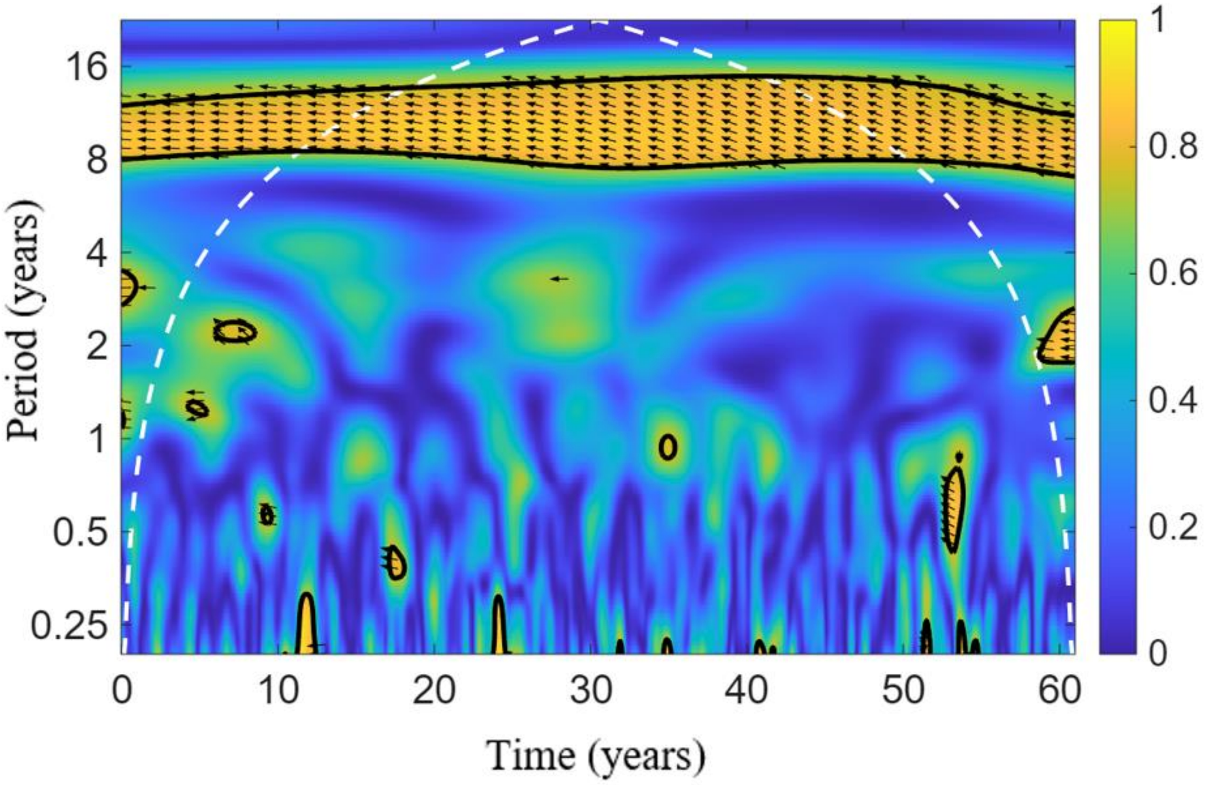

Fig.7. Coherent analysis of sunspot activity and geomagnetic field variations

This figure.7 shows the wavelet coherence between the sunspot number time series and geomagnetic field variations. The diagram reveals localized periods of elevated coherence across multiple time scales, indicating intervals where solar activity and geomagnetic disturbances exhibit synchronized behavior. Significant coherence patches are delineated by contour boundaries.

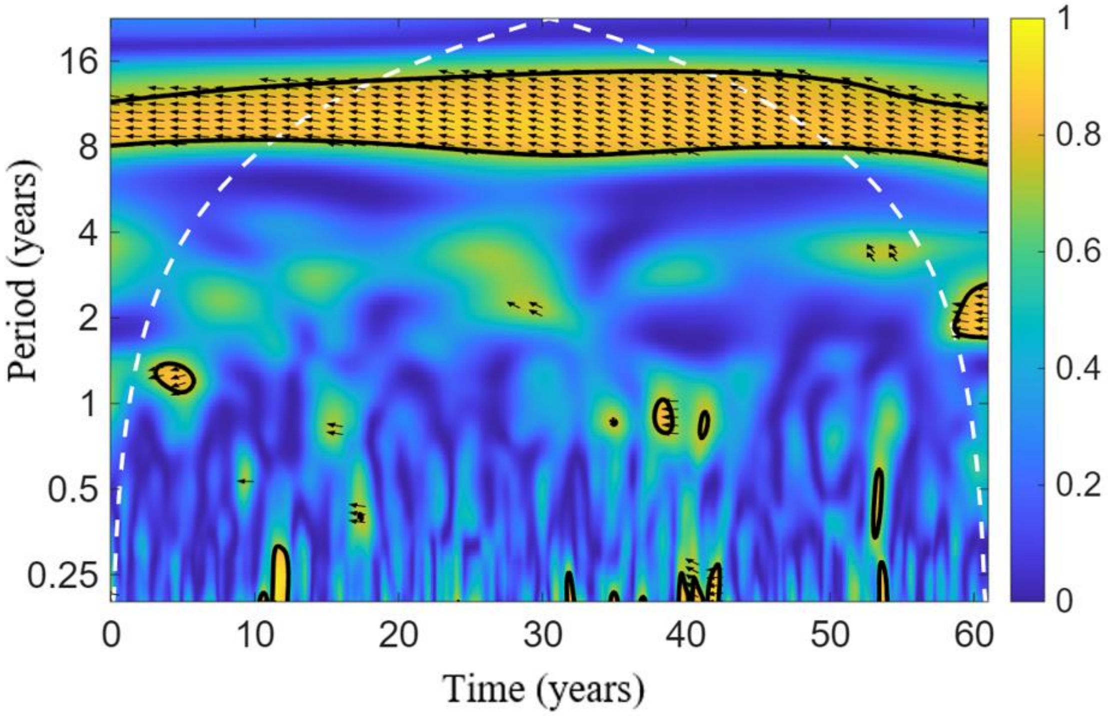

Fig.8. Radio emission from the Sun between 10.7 cm and Dst.

This figure.8 illustrates the coherence between the 10.7-cm solar radio flux and the geomagnetic Dst index. The coherence plot highlights frequency bands where radio emission variability corresponds to geomagnetic storm intensity. High-coherence areas, emphasized by significance contours, indicate time–scale intervals characterized by strong coupling.

Taken together, these results demonstrate that geomagnetic storms emerge precisely when the magnetosphere transitions into a state of maximal synchronization with the intensified interplanetary magnetic field. The persistence and phase uniformity within the high-coherence regions are the necessary and sufficient conditions for efficiently driving the ring current. This finding suggests that forecasting severe space weather should incorporate not only the amplitude of the solar wind parameters but also the degree of sustained phase coherence between the driver and the terrestrial response system.

Conclusion

We initially explored the concept and importance of synchronization through theoretical models. Specifically, we demonstrated frequency synchronization in the van der Pol model and phase synchronization in the Adler equation using numerical experiments. Applying this understanding to real-world data, we used Wavelet Coherence to show a time- and frequency-dependent relationship between solar activity indices (including sunspot numbers and the 10.7 cm radio flux) and the Earth's magnetic field (proxied by the Dst index and IMF B_z components). The highest period of coherence consistently corresponds to the phases of solar maxima, confirming that strong solar activity significantly affects the Earth's magnetosphere. These results confirm that changes in the solar cycle fundamentally determine geomagnetic oscillations and demonstrate a close dynamical coupling between the solar magnetic field and that of the Earth.

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სინქრონიზაციის ეფექტები რთული არაწრფივი სისტემების ურთიერთქმედებისას

მ. მარტიაშვილი, ო. ხარშილაძე, დ. ზილფიმიანი

რეზიუმე

ეს კვლევა იკვლევს სინქრონიზაციის ფუნდამენტურ მექანიზმებს არაწრფივ დინამიურ სისტემებში გარე გავლენის ქვეშ, აკავშირებს თეორიულ ანალიზს რეალურ გეოფიზიკურ აპლიკაციებთან. ჩვენ თავდაპირველად ვამახვილებთ ყურადღებას ვან დერ პოლის არაწრფივ ოსცილატორზე, რათა შევისწავლოთ იძულებითი სინქრონიზაცია, რიცხვითი მოდელირების გამოყენებით გავაანალიზოთ, თუ როგორ ცვლიან გარე პერიოდული ძალები სისტემის ფაზურ და სიხშირულ დინამიკას. მთავარი აღმოჩენა ავლენს, რომ შემთხვევითი ხმაურის შემოღებას, ინტუიციის საწინააღმდეგოდ, შეუძლია წვლილი შეიტანოს რთული ან ქაოტური რეჟიმების რეგულირებაში, რაც იწვევს სტაბილურობის გაზრდას.

ამ ანალიზის მზე-დედამიწის კავშირზე გაფართოებით, ჩვენ ვიყენებთ ტალღური კოჰერენტობის მეთოდს, რათა რაოდენობრივად შევაფასოთ დრო-სიხშირის კოჰერენტობა მზის პარამეტრებს (როგორიცაა $f10.7$ ინდექსი და $IMF\ B_z$ კომპონენტი) და ხმელეთის რეაქციებს შორის. ეს ანალიზი წარმატებით აფიქსირებს მნიშვნელოვან სინქრონიზაციის ეფექტებს, რაც ადასტურებს, რომ ამ სისტემებს შორის კოჰერენტული ურთიერთქმედება პირდაპირ პასუხისმგებელია გეომაგნიტური შტორმების გამოწვევაზე. საერთო ჯამში, ეს ნაშრომი უზრუნველყოფს ყოვლისმომცველ ჩარჩოს სინქრონიზაციის ფენომენების გასაგებად, კონტროლირებადი არაწრფივი სისტემებიდან დაწყებული მასშტაბური ასტროფიზიკური ურთიერთქმედებებით დამთავრებული.

საკვანძო სიტყვები: სინქრონიზაცია, ვან დერ პოლის ოსცილატორი, ვეივლეტ კოჰერენტობა, სტოქასტური რეზონანსი, გეომაგნიტური შტორმები.

Эффекты синхронизации во взаимодействии сложных нелинейных систем

М. Мартиашвили, О. Харшиладзе, Д. Зилпимиани

Резюме

Исследование посвящено фундаментальным механизмам синхронизации в нелинейных динамических системах под внешними воздействиями, объединяя теоретический анализ с практическими геофизическими приложениями. Сначала внимание уделяется нелинейному осциллятору ван дер Поля для изучения вынужденной синхронизации; с помощью численного моделирования анализируется, как внешние периодические воздействия изменяют фазовую и частотную динамику системы. Ключевой результат показывает, что введение случайного шума может, вопреки ожиданиям, способствовать регулированию сложных или хаотических режимов, повышая устойчивость системы.

Расширяя анализ на связь Солнце-Земля, мы применяем метод вейвлет-когерентности для количественной оценки временно-частотной когерентности между солнечными параметрами (такими как индекс F10.7 и составляющая IMF B_z) и земными откликами. Этот анализ успешно выявляет значимые эффекты синхронизации, подтверждая, что согласованное взаимодействие между этими системами напрямую отвечает за запуск геомагнитных бурь. В целом работа формирует комплексную основу для понимания явлений синхронизации от управляемых нелинейных систем до крупномасштабных астрофизических взаимодействий.

Ключевые слова: Синхронизация, осциллятор Ван дер Поля, вейвлет-когерентность, стохастический резонанс, геомагнитные бури.