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Control of Inversions of the Earth's Magnetic Field with Rikitake's Model

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ABSTRACT

Paleomagnetic data demonstrates that the Earth's dipole magnetic field which shields the planet from solar wind, has undergone numerous irregular polarity reversals, with an average rate of approximately 5 per Myr. While many theories exist regarding the cause of these reversals, the most popular linking them to Milankovitch cycles, direct numerical investigation of the core's dynamo is constrained by the extreme parameter values of the Earth's outer core. Consequently, simplified models are essential. One such model is the two-disk Rikitake's dynamo, which is characterized by inherent chaotic inversions, unlike the earlier Bullard's model. Statistical analyses show that the Rikitake's model, much like real paleomagnetic data, exhibits a strong deviation from Poisson statistics. In this study, we modified the Rikitake's system by introducing an external current i_0 corresponding to an external magnetic field. This modification incorporates the hypothesis that the geodynamo is influenced by the weak interstellar magnetic fields found in the Local Interstellar Cloud. Our results show that when the modified Rikitake's model is driven by an external current whose sign changes are synchronized with real paleomagnetic data, the model successfully reproduces the timing of the geomagnetic reversals, supporting the hypothesis of external control of the geodynamo.

Key words: *Earth's magnetic field, Geodynamo, Reversals, Rikitake's Model*

Introduction

The geomagnetic field is essential in protecting Earth from the harmful solar wind and cosmic radiation. It primarily has a dipole structure, with its magnetic moment generally aligned with the planet's rotation axis [1]. Paleomagnetic data confirms that the axial dipole component of the Earth's magnetic field has reversed its polarity many times [2], with these flips occurring in an irregular fashion, shortest interval between reversals are observed to be ~ 1000 years, while longest ones being million years and more. Averaged over the last few million years, the mean reversal rate is approximately 5 per Myr [3]. The reversal process itself shows a notable asymmetry: the field's decay is slower than its return to the opposite polarity. Data also indicates a possibility of a correlation between the field intensity and the time interval between successive reversals [4,5].

The magnetohydrodynamic (MHD) dynamo hypothesis, while still containing unresolved aspects, is the generally accepted theory for the generation of the Earth's magnetic field. The Earth's core consists of a solid inner core and a liquid outer core. The outer core is an electrically conducting fluid, and the convective motion of this liquid metal, creating circular electric currents, drives the dynamo action. The resulting field is predominantly a dipole, with its axis closely aligned with the Earth's axis of rotation. However, to initiate this field generation process, an initial magnetic field is required. Potential sources for this initial field include the gyromagnetic effect, where a rotating body spontaneously develops magnetization along its rotation axis or, alternatively, the influence of the external solar or interstellar magnetic fields.

The random nature of the reversals is attributed to the fact that, given the parameters of the Earth's core, the geodynamo operates close to an instability regime. In this state, a minor external perturbation can cause a transition from a stable to a chaotic regime (leading to reversals). In the same way, a small, deterministic external influence can also lead to the suppression of the chaotic regime, forcing a transition back to a stable mode, a key method in the control of chaotic systems.

Numerical simulations of the geodynamo face significant difficulty because they cannot accurately match the extreme fluid parameters of the Earth's outer core, such as a magnetic Prandtl number $\sim 10^{-5}$ [1]. Consequently, simplified models are often used to explore the mechanisms of field reversals.

Statistical analysis of the paleomagnetic data provides key insights into the geodynamo's behavior. Sorriso-Valvo et al. [6] showed that the sequence of polarity reversals strongly deviates from simple Poisson statistics. This deviation is attributed to temporal clustering, suggesting the presence of long-range correlations and memory effects in the underlying process. Consolini and De Michelis [7] offered evidence that these reversals may be a stochastic resonance phenomenon, identifying connections between reversal residence times and the Earth's orbital eccentricity variation (~ 0.1 Myr). There is also more evidence given from data linking geomagnetic reversals to other Milankovitch cycles [8]. There are experiments like "DRESDYN" aiming to investigate precession as a source of dynamo action [9].

Runcorn demonstrated that an internal vortex magnetic field is generated within the Earth's core during its rotation. Furthermore, he showed that the MHD equations theoretically permit the inversion of the core's magnetic moment, providing the first mathematical proof of magnetic field reversal [10]. The first physical realization of this problem, however, was provided by Rikitake. The two-disk Rikitake model is a well-known system used to investigate reversals because it inherently produces chaotic reversals. Statistical work [6] has demonstrated that the Rikitake model's reversal sequence also shows strong evidence of clustering, mirroring the non-Poissonian behavior of real data. Our work presents results showing that the Rikitake's model can reproduce real reversal data patterns when an external driver is applied to the system during these reversals.

Rikitake two disc model

Bullard [11] established a single-disk model in 1955, which was based on Faraday's effect, where a moving conductive disk created a magnetic field. Although it was possible to record changes in the magnetic field in Bullard's model, magnetic reversals were not observed in this model. Tsuneji Rikitake [12], however, developed Bullard's idea in 1957 and hypothesized that each convective cell in the liquid core could be represented by a solid conductive disk. His model is a two-disk system where the disks rotate within the magnetic field created by the current flowing in the other disk . It is characterized by inherent chaotic polarity inversions, mimicking real paleomagnetic data.

By considering Faraday's Law and Kirchhoff's Voltage Law, we obtain a system of differential equations:

$$L_1 \frac{dI_1}{dt} = -R_1 I_1 + \Omega_1 M I_2 \tag{1}$$

$$L_2 \frac{dI_2}{dt} = -R_2 I_2 + \Omega_2 N I_1 \tag{2}$$

$$C_1 \frac{d\Omega_1}{dt} = G_1 - M I_1 I_2 \tag{3}$$

$$C_2 \frac{d\Omega_2}{dt} = G_2 - N I_1 I_2 \tag{4}$$

Where I_1 and I_2 denote the current passing through the first and second conductors, and ω_1 and ω_2 are the angular velocities of the disks. L represents the inductance. R is the resistance, and M is the mutual inductance. C_1 and C_2 are the moments of inertia of the disks, and G_1 and G_2 are the torques acting on them. For simplicity:

$$L_1 = L_2 \equiv L, \quad R_1 = R_2 \equiv R, \quad M = N, \quad C_1 = C_2 \equiv C, \quad G_1 = G_2 \equiv G$$

To simplify the numerical calculations and reduce the number of parameters, we transition to dimensionless variables:

$$I_1 = \sqrt{\frac{G}{M}} i_1, \quad I_2 = \sqrt{\frac{G}{M}} i_2, \quad \Omega_1 = \sqrt{\frac{GL}{CM}} \omega_1, \quad \Omega_1 = \sqrt{\frac{GL}{CM}} \omega_1, \quad t = \sqrt{\frac{CL}{GM}} \tau$$

Thus, equations (1), (2), (3), and (4), using dimensionless variables, are written in the form of the following system of differential equations:

$$\begin{cases} \frac{di_1}{d\tau} = -\mu i_1 + \omega_1 i_2 \\ \frac{di_2}{d\tau} = -\mu i_2 + \omega_2 i_1 \\ \frac{d\omega_1}{d\tau} = 1 - i_1 i_2 \\ \frac{d\omega_2}{d\tau} = 1 - i_1 i_2 \end{cases} \tag{5}$$

Where $\mu = R\sqrt{\frac{C}{LMG}}$

According to Cook and Roberts [13] (1990), the Rikitake's system is characterized by two distinct time scales $\tau_m = \frac{CR}{GM}$ ("mechanical time scale") and $\tau_e = \frac{L}{R}$ ("electromagnetic diffusion time"), The dimensionless time is denoted by τ.

The electromagnetic diffusion time, τ_e, can be interpreted as the decay time of the magnetic field in the core. It is given by the formula - $\tau_e = \frac{L^2}{\nu_m}$ [14], where L is the characteristic scale of the outer core $\sim 2000\ km$ and magnetic diffusivity $\nu_m \approx 2\ m^2/s$ [1], based on these typical τ_e can be estimated as $\sim 10^4$ years.

Cook and Roberts [13] treat the mechanical time scale, τ_m, as a time required for the slow Alfven waves crossing the core, and they give formula $\mu = \sqrt{\frac{\tau_m}{\tau_e}} \sim 10^{-3} - 10$ for earth.

For our research we take values of $\tau_m = 10^4, \mu = 1$ and therefore the dimensionless time unit is equivalent to $\tau = \sqrt{\tau_e \tau_m} = 10^4$ years.

Result of (5) for a certain set of parameters are shown in Fig. 1.

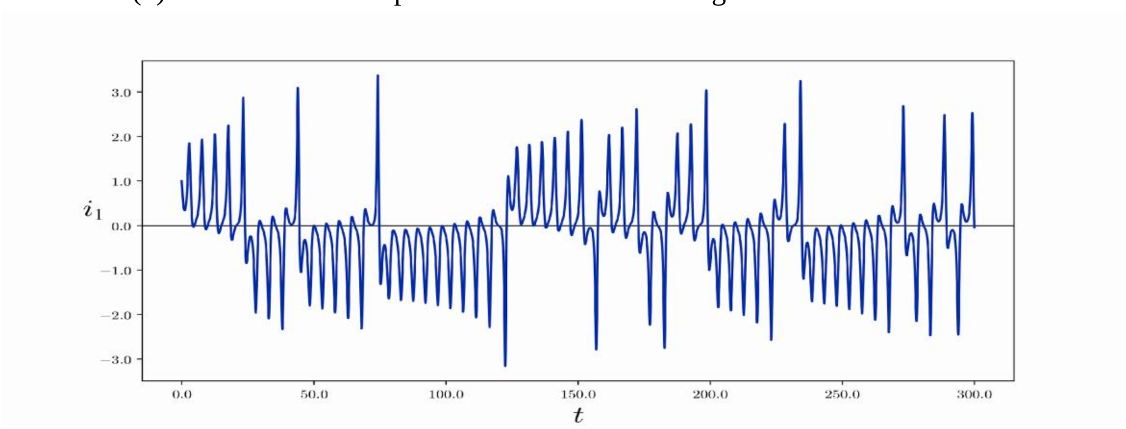

Fig. 1. Result of Rikitake's system for parameters - $\mu = 1,\ i_1 = 1,\ i_2 = -1,\ \omega_1 = 0,\ \omega_2 = 1$

New System

Now we want to study the behavior of the system when some external factor an external magnetic field acts on it. The effect of the external factor is expressed by the induced current i_0. The modified system will take the following form:

$$\begin{cases} \dfrac{di_1}{d\tau} = -\mu i_1 + \omega_1(i_2 + i_0) \\ \dfrac{di_2}{d\tau} = -\mu i_2 + \omega_2(i_1 + i_0) \\ \dfrac{d\omega_1}{d\tau} = 1 - i_1(i_2 + i_0) \\ \dfrac{d\omega_2}{d\tau} = 1 - (i_1 + i_0)\, i_2 \end{cases} \tag{6}$$

Here i_0 is determined by the external magnetic field. The Earth and the Solar System are constantly moving through the galaxy, where many types of magnetic fields are encountered. It is possible that the external magnetic field acting on our system is the interstellar magnetic field found within the Local Interstellar Cloud. In order to estimate characteristic values for the induced current, i_0, and the system currents, i_1 and i_2, we must use Ohm's Law. Ohm's Law states that current density, j, is proportional to the product of conductivity, σ, velocity, v, and magnetic field, B, i.e., $j \propto \sigma v B$, Since the current i is proportional to the current density, j, a reasonable estimate requires using the characteristic values for conductivity, velocity, and magnetic field strength for both the Earth's outer core and the interstellar medium. For the Earth's inner core these parameters are:

$\sigma_1 \sim 10^{14} - 10^{16}\ [s^{-1}]$ [15], $v_1 \sim 10^{-2}\ cm/s$ [16], $B_1 {\sim} 2.5 \cdot 10\ Gauss$ [17]

In the interstellar medium conductivity is given by the formula [18] :

$$\sigma {\sim} 6.5 \times 10^6 T^{3/2} s^{-1} \tag{7}$$

And the temperature, $T \sim 10^3 - 10^4 K$ [19], therefore for the interstellar medium:

$\sigma_1 \sim 6.5 \cdot 10^{10} - 6.5 \cdot 10^{12}\ [s^{-1}]$, $v_1 \sim 2.5 \cdot 10^6\ cm/s$ [20], $B_1 {\sim} 10^{-5}\ Gauss$ [2]

We found that $\frac{j_1}{j_0} {\sim} 1 - 10^2$, and currents, i_0 and i_1, may differ by one or two orders of magnitude

Results

We conducted numerical calculations on the modified system for individual i_0 - s. For a constant i_0, we found that after a certain amount of time, the currents stabilize at a constant value. If the magnitude of the external current is smaller, it takes a longer time to stabilize at the constant value. If the i_0 signal represents a rectangular wave, the numerical calculations show that inversions in the system occur in correspondence with the rectangular wave (Fig.2, b). We obtain a similar result when adding a sinusoidal signal as well (Fig. 2, a).

Besides well-known functions, we chose to incorporate a signal corresponding to the geomagnetic field reversals themselves to investigate the system's response. Panel (b) in Fig 3. displays the polarity reversals for the last 3.5 Myr, while Panel (a) presents the results from the modified Rikitake's system. In this modified model, the external current i_0 is represented by a rectangular signal that switches polarity in accordance with the real geomagnetic reversal data [2]. The signal is defined with a constant amplitude of ± 1 and includes additive weak white noise, simulating a realistic external perturbation. The results clearly demonstrate that the modified model changes sign at approximately the same moments in time as the actual paleomagnetic observations.

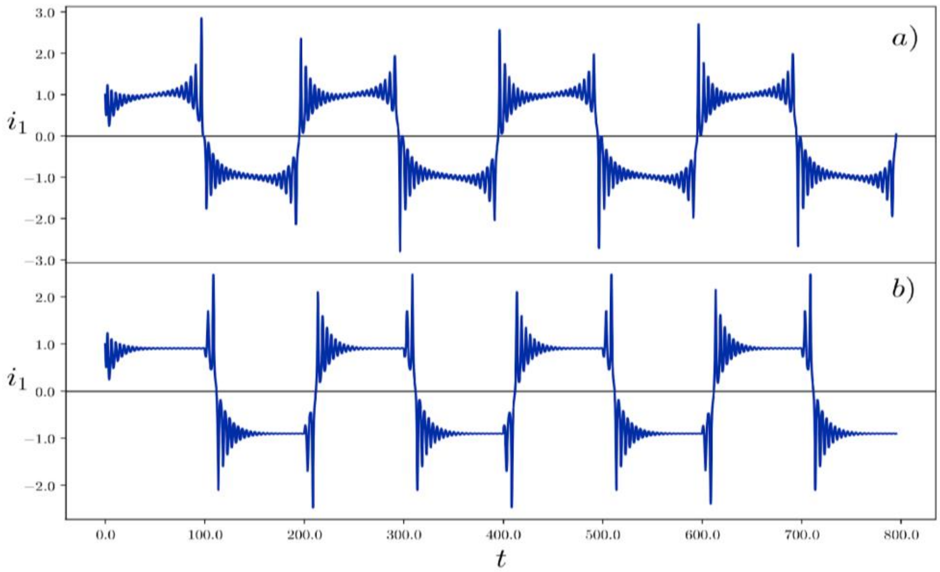

Fig. 2 Response of the modified Rikitake's system with parameters $\mu = 1$, $i_1 = 1$, $i_2 = -1$, $\omega_1 = 0$, $\omega_2 = 1$. Panel (a) presents the system's behavior when the external current i_0 is a sinusoidal signal, and Panel (b) displays the results obtained using a rectangular signal for i_0.

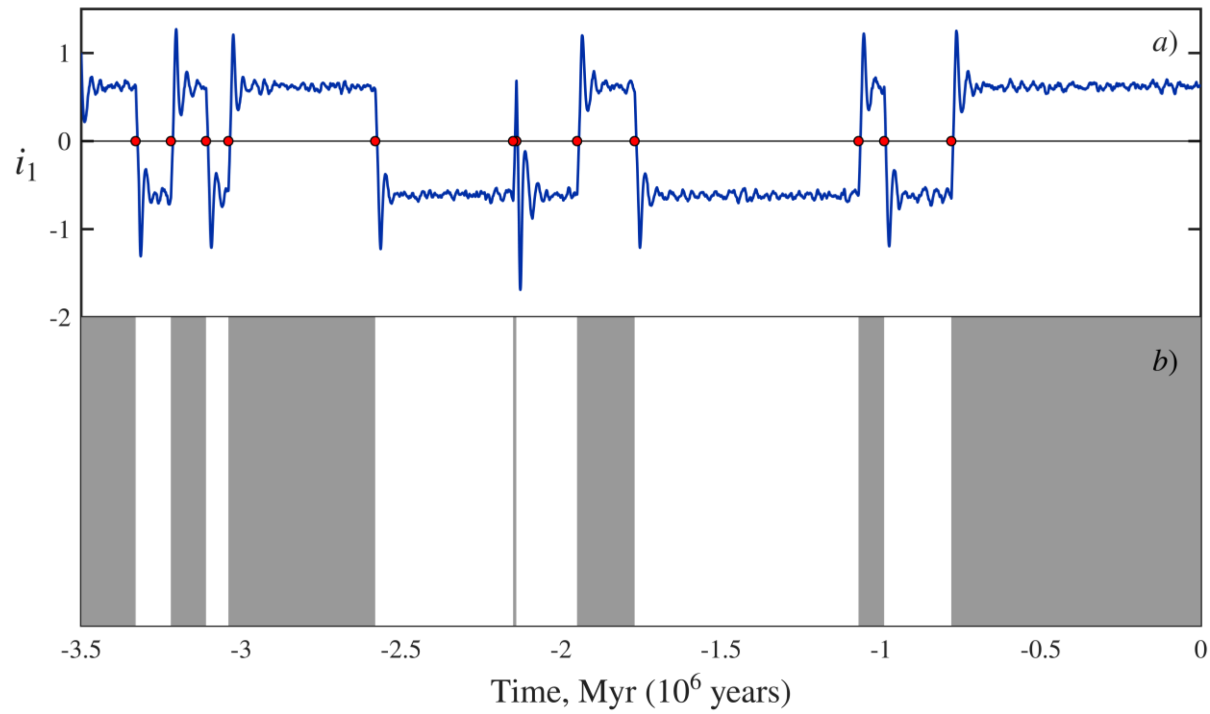

Fig. 3 Simulation results of the modified Rikitake's dynamo and comparison with paleomagnetic data. The system is driven by a external current (rectangular signal) whose polarity is dictated by the real reversal record, potentially mimicking the effect of the interstellar magnetic field. Panel (a) shows the output of the modified Rikitake's system. Panel (b) displays the paleomagnetic reversal data, where the gray areas indicate the current (normal) polarity of the Earth's magnetic field, and the white areas showcase reversed polarity.

Conclusions

In this article, we investigated the Rikitake's two-disk dynamo model, which is frequently utilized to describe the Earth's magnetic field dynamics. Unlike the classical version of the model, we explored a modified Rikitake's system by introducing an external parameter: an external current i_0.

Following the estimation of the characteristic order of this external parameter, we subjected the system to various forms of i_0 and observed its subsequent behavior. The application of a constant external current drove the system's internal currents toward a stable, constant value. Conversely, the introduction of a sinusoidal or rectangular wave induced polarity inversions within the system that synchronized with the changes in the external current.

Beyond using standard mathematical functions, we introduced an external current that directly correspons to the time series of known geomagnetic field reversals [2]. The result demonstrated that the system's polarity inversions occurred in agreement with the paleomagnetic data. This suggests that if the interstellar magnetic field in space changes according to a specific rule, it can significantly influence the geodynamo's behavior and potentially control the timing of magnetic field reversals.

Overall, we conclude that by modifying the Rikitake's model and incorporating an external factor, it is possible to control and synchronize the inversions within the system. This hypothesis that the geodynamo may be controlled by the Local Interstellar Cloud's magnetic fields opens up an interesting question for future research, that needs further theoretical analysis and numerical investigation.

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დედამიწის მაგნიტური ველის ინვერსიების კონტროლი რიკიტაკის მოდელის გამოყენებით

ო. ხარშილაძე, ლ. წულუკიძე, ლ. კეზუა, დ. ზილფიმიანი

რეზიუმე

პალეომაგნიტური მონაცემები აჩვენებს, რომ დედამიწის დიპოლურ მაგნიტურ ველს, რომელიც იცავს პლანეტას მზის ქარისგან, წარსულში მრავალი არარეგულარული პოლარობის ინვერსია (შებრუნება) ჰქონდა, საშუალო სიხშირით დაახლოებით 5 ინვერსია მილიონ წელიწადში (Myr). მიუხედავად იმისა, რომ არსებობს მრავალი თეორია ამ ინვერსიების გამომწვევ მიზეზებთან დაკავშირებით, რომელთაგან ყველაზე პოპულარული მათ გარე ზემოქმედებას, მაგალითად, მილანკოვიჩის ციკლებს, უკავშირებს, დედამიწის გარე ბირთვის ექსტრემალური პარამეტრები ზღუდავს დინამოს პირდაპირ რიცხვით კვლევას. შესაბამისად, გამარტივებული მოდელები აუცილებელია. ერთ-ერთი ასეთი მოდელია ორდისკიანი რიკიტაკის დინამო, რომელიც, ადრინდელი ბულარდის მოდელისგან განსხვავებით, ხასიათდება შინაგანი ქაოსური ინვერსიებით. სტატისტიკური ანალიზი აჩვენებს, რომ რიკიტაკის მოდელი, ისევე როგორც რეალური პალეომაგნიტური მონაცემები, მნიშვნელოვნად გადახრილია პუასონის სტატისტიკიდან. ამ კვლევაში ჩვენ შევცვალეთ რიკიტაკის სისტემა გარე დენის (i_0) შემოტანით, რომელიც გარე მაგნიტურ ველს შეესაბამება. ეს მოდიფიკაცია ეფუძნება ჰიპოთეზას, რომ გეოდინამოზე გავლენას ახდენს ადგილობრივ ვარსკვლავთშორის ღრუბელში (Local Interstellar Cloud) არსებული სუსტი ვარსკვლავთშორისი მაგნიტური ველები. ჩვენი შედეგები აჩვენებს, რომ როდესაც მოდიფიცირებული რიკიტაკის მოდელი იმართება გარე დენით, რომლის ნიშნის ცვლილებაც სინქრონიზებულია რეალურ პალეომაგნიტურ მონაცემებთან, მოდელი წარმატებით იმეორებს გეომაგნიტური ინვერსიების დროს, რაც მხარს უჭერს გეოდინამოს გარე მოდულაციის ჰიპოთეზას.

საკვანძო სიტყვები: დედამიწის მაგნიტური ველი, გეოდინამო, რევერსალები, რიკიტაკეს მოდელი

Контроль инверсий магнитного поля Земли с помощью модели Рикитаки

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Резюме

Палеомагнитные данные демонстрируют, что дипольное магнитное поле Земли, которое защищает планету от солнечного ветра, претерпело многочисленные нерегулярные инверсии полярности со средней частотой около 5 инверсий на миллион лет. Хотя существует множество теорий относительно причин этих инверсий, самая популярная из которых связывает их с внешним воздействием, например, с циклами Миланковича, прямое численное исследование динамо-процесса в ядре ограничено экстремальными значениями параметров внешнего ядра Земли. Следовательно, упрощенные модели имеют важное значение. Одной из таких моделей является двухдисковое динамо Рикитаки , которое, в отличие от более ранней модели Булларда, характеризуется внутренними хаотическими инверсиями. Статистический анализ показывает, что модель Рикитаки, как и реальные палеомагнитные данные, демонстрирует сильное отклонение от статистики Пуассона. В данном исследовании мы модифицировали систему Рикитаки, введя внешний ток i_0, соответствующий внешнему магнитному полю. Эта модификация включает гипотезу о том, что на геодинамо влияют слабые межзвездные магнитные поля, обнаруженные в Локальном межзвездном облаке. Наши результаты показывают, что когда модифицированная модель Рикитаки управляется внешним током, смена знака которого синхронизирована с реальными палеомагнитными данными, модель успешно воспроизводит время геомагнитных инверсий, что подтверждает гипотезу о внешнем модулировании геодинамо.

Ключевые слова: Магнитное поле Земли, геодинамо, инверсии магнитного поля, модель Рикитаки