

## **Simulation of point explosion's seismic energy by means of the frequency spectrum of body waves**

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### ***Abstract***

Solve the inverse problem, which is aimed at modeling a discrete frequency spectrum of seismic body waves generated by artificially weak point explosion or a natural earthquake ( $M \leq 4$ ). Proposed a spherical model of the hollow area of the point explosion and used a well-known analytical method for modeling the hydro-mechanical oscillations of a liquid drop. Innovation in the applied work is the use of a complete solution of the radial Euler equation. Such a modification of the classical scheme, which uses only an internal solution is mathematically quite correct, because it means virtuality of seismic source's elastic oscillation. As a result, with the help of the discrete spectrum of seismic body waves can be determined the linear parameters and total energy of point explosion (weak earthquake) that is approximated as a hollow body with spherical shape.

*Keywords: weak earthquake, hollow area, elastic oscillation, fundamental frequency.*

**Introduction.** Despite the similarity of effects that accompany earthquakes and explosions, these phenomena have a different nature. Primarily, the point source of explosion can be located anywhere while hypocenter of earthquake is always in seismically active zone and significant tectonic stresses area. The energy released during an earthquake and a point explosion in the focal zone is distributed according to the same pattern from a mechanical point of view: a significant portion of this energy expended in irreversible changes in the sources area, the rest part of its generates the body and surface seismic waves. The point explosion's energy comparable with the weak earthquakes ( $M \leq 4$ ) (perhaps with the exception of particularly powerful nuclear explosions), during which most of the energy of elastic deformation is consumed to generate the seismic body waves. There are differences in the method of measuring the total energy released during moderate and strong earthquakes and point explosions. However, before the era of underground nuclear explosions was assumed that in case of lack of dispersion, the amplitude of the elastic body waves of point explosion should be proportional to the square root of the oscillations energy density (as for earthquake). However, analysis of the underground nuclear explosions's series data in 1958, that in fact are analogous to the point explosions, this is not confirmed. The relationship between the explosion's energy and the amplitude was linear, so new model of explosion source have been proposed [1]. According to this model, the area of any explosion source is a special, so-called zone

violation, in which energy is transferred and absorbed by nonlinear laws. It is believed that the radius of the zone is proportional to the cube root of the energy of the explosion. Further, beyond the violations zone, disturbance is linearly elastic in the medium. It is assumed that propagating compression wave here are not subjected to the dispersion. In this case, the individual harmonics of the body waves can propagate independently. Obviously, such a situation is ideal and is expected wave propagation without any obstacles, which is possible only in a completely homogeneous medium. In fact, analysis of seismic waves propagation can be quite complex in the elastic medium, not only because of the heterogeneity of the environment and the absorption of waves, but also the existence of a free boundary, for example: the earth's surface and the fault. Therefore, it was suggested that the medium is uniform in a sufficiently large area, the radius of which substantially exceeds the radius of the explosion energy release, in other words, the zone of nonlinear transformations. In some studies, factually the model in [1] has been used to simulate the change of the radial stress at the interface of the inelastic and elastic region [2].

**Model of a point explosion.** The purpose of work [1] was an analytical justification of the relationship between soil displacement and the explosion energy carried out by body seismic waves. Obviously, such a task is straightforward because suggests a correlation between the known parameters: given energy of the explosion and the observed value of soil displacement. However, besides displacements soil informative are also themselves seismic wave frequency spectrum, which is also dependent on the energy of the explosion. If we consider that the heterogeneity of the medium influenced less significant effect on seismic waves, in particular, on their frequency, an attempt to study the relationship between the explosion energy and body waves can be considered as a perspective task. It should be noted that assuming no dispersion of body waves, in contrast to [1], it is possible to solve the inverse problem, i.e. determine the explosion energy by body waves frequency spectrum. An attempt to solve such a problem has been undertaken in the work [3]. Physical conditions of the work actually coincide with the statement of the problem in [1] as it also assumes that the point explosion is followed by an avalanche-like release of energy resulting in the generation of shock waves and plastic deformations. So there should be a zone of non-linearity. On the border of this zone must establish a balance between the pressure force and the elastic force of the environment. It is evident that the balance of forces arise only if the energy density of the explosion will be commensurate with the energy density of the elastic deformation. The front propagation of disturbances caused by a point explosion is supposed to be radial in a homogeneous incompressible medium. Spherical symmetry is violated with increasing distance in an inhomogeneous medium. However, in any case, the deviation from sphericity is unlikely to be so significant that the boundary of the plastic deformation of the explosion could not be approximated by a spherical, or more complex, by the surface of the rotation ellipsoid. The same form can be assumed also for the outer boundary of linear elasticity area, if we neglect the inhomogeneity of the medium and to postulate the absence of free boundaries. In this case, in work [3] was proposed hypothesis that body seismic waves are the result of self mechanical oscillations of the body, which is an spatial abstraction of the linearity zone around the point explosion source. Obviously, these oscillations induced by the elastic force and should have a discrete frequency spectrum. In the simplest case, discussed body may have the shape of a hollow sphere, the inner radius equal to the radius of the zone violation. The next step is to approximate the area of a point

explosion by hollow rotation ellipsoid. This figure has a lower degree of symmetry comparable with the sphere. Due to this the degenerate oscillations will take place. Therefore effects of shear deformation can be neglected at a point explosion, unlike earthquakes. This is essential since simulation of the vibrational spectrum, generated by a strong earthquake is very challenging. During the earthquake, as a rule, there are considered as body waves, so surface seismic waves. Analytical solution of this problem is obtained only in the case of spherical symmetry by presenting the shear modulus as a sum of normal modes of vibrations [4].

**Statement of the problem.** The problem of accurate determination of the total energy released by an earthquake, is the main task of seismology. It is known that some of the total energy, the so-called seismic energy, is consumed to generate the body and surface seismic waves. Unlike natural earthquakes, the total energy is known in advance during point explosion. Furthermore, for an underground nuclear explosions has been determined that the approximately 5-8% of the total energy passes to the elastic seismic waves [5,6]. Therefore, if the total energy of the explosion is known, there is no need to determine the seismic energy by seismic data. However, estimation of total energy, during the earthquake, is a problem. It is obvious that the direct transfer of the result, which is valid for explosions, to an earthquake is incorrect. It is known that seismic energy is dependent on the source volume [7]. In the case of moderate and strong earthquakes this area is difficult to determine because of the considerable scatter of aftershocks in time and space. Therefore, proportion of seismic energy in the total energy can very substantially changed with increasing earthquake magnitude. However, for small earthquakes, like an nuclear and technical explosions, almost all the seismic energy transfer in body waves. Therefore, the relationship between seismic energy and the total energy of the earthquake in both cases varies in the same range. It is obvious that, like explosions, for small earthquakes seismic source zone is a violation within the meaning of [1].

Assessment of the explosion volume, and consequently, the seismic energy, when we know its total energy, it is not difficult. During the earthquake the total energy is unknown that could be estimated by seismic data. Therefore, any attempt that simplifies the tedious process of seismic source volume determination can be considered relevant. In particular, it appears that the volume of weak earthquakes source can be quite easily define following [3], the essence of which is given below.

**Scheme of mathematical modeling.** The physical analogy with hydromechanical natural oscillations of a liquid incompressible drop of spherical configuration has been used as the basis of the model proposed in [3]. It is known that small perturbations of its surface is able to maintain the shape of a drop due to the action of capillary forces [8]. If it is assumed that the generation of body waves associated with perturbations of the boundary of a hollow elastic body, we can use the mathematical scheme of self hydro-mechanical oscillations of a spherical drop. As it was shown in [8] this scheme can be generalized to the case where the drop has shape of elongated rotation spheroid. Obviously, for the area of a point explosion such shape is more appropriate than a sphere. Though, to estimate the volume of seismic source is enough to use the result, which corresponds to the most simple spherical symmetry. According to [8], a mathematical model is correct if the amplitude of the oscillation or radial displacement of the boundary of the hollow body modeling area of point explosion considerably small compared with the characteristic linear

dimension of the body. Additionally, the oscillation speed of the hollow body surface should be substantially less than the velocity of shock waves generated in the nonlinear transformation zone or in seismic source area. It is obvious that both of these requirements are performed that is the necessary condition for small perturbations causing the linear elasticity, and generating self mechanical oscillation of the body, identified with the area of the point explosion. It is believed that regardless of the site condition, this area despite a small radial displacement of its borders is incompressible and homogeneous, both before and after an explosion (or a weak earthquake). Consequently, the oscillation motion of the hollow body obeys the Laplace equation

$$\Delta\psi = 0, \quad (1)$$

where  $\psi$  is potential, oscillation speed -  $\vec{V} = grad\psi$ .

In a spherical coordinate system, the condition of equilibrium boundary of the hollow body, modeling area of a point explosion, is given with the Laplace formula for a liquid drop

$$P_1 - P_2 = KL \left( \frac{1}{R_1} + \frac{1}{R_2} \right), \quad (2)$$

where  $P_1$  and  $P_2$  are the pressures respectively inside and outside the sphere,  $R_1$  and  $R_2$  are the principal curvature radii of the oscillating surface. The coefficient of capillary surface tension giving the elasticity effect is replaced by the product of the uniform compression modulus  $K$  by the characteristic linear dimension of the sphere boundary  $L$

$$K = \frac{E}{3(1-2\sigma)}, \quad (3)$$

where  $E$  is the tension modulus (Young's modulus) and  $\sigma$  - the Poisson coefficient.

Therefore the difference between the pressures can be defined by means of the expression [8]

$$\Delta P = P_1 - P_2 = -\rho g u - \rho \frac{d\psi}{dt}, \quad (4)$$

where  $u$  is the radial displacement producing the oscillation of the spherical surface,  $\rho$  - density of the medium,  $g$  - gravity force acceleration. In the Spherical system, the displacement velocity is related with the motion potential by the expression

$$\frac{\partial u}{\partial t} = \frac{\partial \psi}{\partial r}. \quad (5)$$

Since the gravity force does not influence the elastic deformation effect without loss of generality in (4), we can neglect the first term on the right-hand side, i.e. in the sequel it will be assumed that  $\Delta P = -\rho \frac{\partial \psi}{\partial t}$ , which coincides in form with the first motion integral [8].

This condition and also analytical expression for the parameter  $\frac{1}{R_1} + \frac{1}{R_2}$  defined by variation surface used in the expression (4), from which we obtain the boundary condition for  $\psi$

$$\frac{\partial^2 \psi}{\partial t^2} \Big|_r - \frac{K \cdot L}{\rho r^3} \left\{ 2 \frac{\partial \psi}{\partial r} + \frac{\partial}{\partial r} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial r} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 \psi}{\partial \theta^2} \right] \right\} = 0. \quad (6)$$

Condition (6) is valid if the potential will have the form of a standing wave

$$\psi = A \cdot F(r, \theta, \varphi) e^{i\omega t}, \quad A = \text{const}, \quad (7)$$

where the function  $F(r, \theta, \varphi)$  satisfies the Laplace equation (1). As is well known, any solution of this equation can be represented as a linear combination of volumetric spherical functions:

$F(r, \theta, \varphi) = X(r)Y(\theta)Z(\varphi)$  where  $X(r)$ - radial function,  $Y(\theta) = P_n^m(\cos\theta)$ , - the Legendre function,  $Z(\varphi) = e^{im\varphi}$ . In the work [3] is used the function:  $X = A_n r^n + B_n r^{-(n+1)}$ , that is full solution of the Euler radial equation. This is different fundamentally from the solution in [8], where the potential in (7) represented only by the inner part of the radial function  $\sim r^n$ . As a result, after standard transformations, the discrete spectrum of natural frequencies of a hollow sphere can be obtained

$$\omega_n^2 = \frac{KL}{\rho r^3} \left[ (n-1)(n+2) \frac{nA_n r^{n-1} - (n+1)B_n r^{-(n+2)}}{A_n r^{n-1} + B_n r^{-(n+2)}} \right], \quad (8)$$

where  $n \geq 2$  ( $n = 0$  corresponds to a state at rest,  $n = 1$  to translational motion).

Expression (8) differs from the formula defining oscillations of a solid spherical drop. First, in (8) is presented as a multiplier value of compression modulus and density of the medium, which is the square of the velocity of body waves [3]. Addition, solution for a solid sphere must be valid everywhere, including the focal point. So in classical scheme only the inner solution of the Euler equation is used as a solution of the form:  $r^{-(n+1)}$  at point  $r = 0$  is divergent. For a hollow body this problem does not exist and therefore we can use a general solution. As a result, as is seen from (8), the frequency should not be real, it may be has an imaginary value too. Such a situation is quite favourable when modelling the point explosion region whose inner surface must bound the volume in which shock waves propagate and plastic deformations take place. An example of a relatively simple formula for a hollow sphere of finite thickness clearly shows that it is only by means of a general solution of the Euler equation that we can define the size of the region where elastic forces may generate seismic waves. For this purpose, it is necessary to determine the constants of a complete radial function by introducing a radius of the boundary between the linear and nonlinear zones. On this surface, according to the model, the frequency of elastic waves could be zero. Thus, from (8) we have

$$nA_n R^{n-1} = (n+1)B_n R^{-(n+2)}, \quad (9)$$

which implies

$$B_n = \frac{n}{n+1} A_n \cdot R^{2n+1}$$

Consequently, because of the relation (9) constants in the formula (8) should be excluded. After the introduction of the parameter  $R$ , the characteristic linear dimension of the hollow body also be defined:  $L = (r - R)$ . Finally the following expression for discrete frequency spectrum has been obtained

$$\omega_n^2 = \frac{\alpha V_m^2}{r^2} \left[ (n-1)(n+2) \frac{\left(\frac{r}{R}\right)^{n-1} - \left(\frac{R}{r}\right)^{n+2}}{\frac{1}{n}\left(\frac{r}{R}\right)^{n-1} + \frac{1}{n+1}\left(\frac{R}{r}\right)^{n+2}} \right], \quad (9)$$

were  $\alpha = (1-R/r)$ ,  $V_m = \left(\frac{K}{\rho}\right)^{\frac{1}{2}}$  - velocity of longitudinal body wave without taking into account

the shear deformation. Velocity of primary seismic wave:  $V_p = \left[\frac{E}{3\rho(1-2\sigma)} + \frac{2E}{3\rho(1+\sigma)}\right]^{\frac{1}{2}}$ . It almost

exceeds the speed of secondary shear wave  $V_s = \left(\frac{G}{\rho}\right)^{\frac{1}{2}}$  average by a factor 1.7 in any environment

( $G = \frac{E}{2(1+\sigma)}$  -transverse shear modulus). So velocity  $V_m \approx 0.8V_p$ .

**Discussion of the inverse problem.** The aim of this work is to show the way for a relatively simple solution of the inverse problem for the hollow sphere approximation, when two linear parameter of a point explosion were defined: the radii of the plasticity and linear elasticity zones. Besides the physical parameters of the considered medium it is assume that the spectrum of point explosion frequencies is also the given one, then we can define the unknowns  $R$  and  $r$ . In fact, the problem is in the knowledge of the fundamental frequency  $\omega_2$  ( $n=2$ ), as in a homogeneous medium  $\frac{\omega_3}{\omega_2} \approx 2$ . Therefore, the spectral analysis of point explosion is necessary to solve the inverse problem, which should give the value of the fundamental frequency of seismic body waves. The desired radius can be determined by the first two equations (9) corresponding frequencies  $\omega_2$  and  $\omega_3$ .

A distinctive feature of this work is to neglect of the effect of shear deformation and use the modulus for compression as a parameter of the elastic properties of the Earth's environment. This assumption is justified in the case where the power of a point explosion is not higher than the power of weak earthquakes ( $M \leq 4$ ). According to [2], for sufficiently powerful underground nuclear explosions, when the transverse seismic waves became visible, their fundamental frequency is given by equation

$$\omega_0 = \frac{2V_s}{R}, \quad (10)$$

where  $R$  is the radius of the surface elastic wave generation. Obviously, in the case of the expression (9) yields the fundamental frequency of the same order as that of the expression (10).

It is notable that one solution of task is known to determine the frequency spectrum of the radial natural oscillations of an elastic sphere. Physically, it is obvious that realization such oscillations is possible only when the speed of displacement changes is directed along a radius and depends only on coordinate  $r$  [8,10]. According to the boundary condition on the surface of the sphere, the radial component of the strain tensor is equal zero. The problem of periodic oscillations

in time reduces to the general wave equation for the potential movement. The solution of this equation, which is valid throughout the volume of a sphere, including its centre, has the form

$$\varphi = A \frac{\sin kR}{R} e^{-i\omega t} \quad (11)$$

Due to equation (11) and the boundary conditions resulting from the Hooke's condition on the surface  $R = r$ , the transcendental equation was obtained for  $kr$

$$\frac{tgkr}{kr} = \frac{1}{1 - 0.25 \left[ \left( \frac{V_p}{V_s} \right) kr \right]^2} \quad (12),$$

$k \sim \frac{1}{r}$  - is the wave number. The roots of this transcendental equation (the exact analytical solution of which is impossible) determine the frequency of natural oscillations of an elastic sphere. However it is possible approximate analytical or numerical solution, after which the fundamental frequency of the radial oscillations of an elastic sphere is determined by the velocity of the longitudinal wave:  $\omega_k = V_p k$ .

Thus, using the formulas (9) and (10), as well as the numerical solution of equation (12) when the velocity ratio is 1.7, it is possibility to compare different models of elastic oscillation of point explosion. The radius value of linear zone should be considered as the core defined by the expression (9). The underground nuclear explosions represent the most convenient case as all parameters are known for comparative analysis. It is suitable, for example, the very first underground nuclear explosion conducted in Nevada in 1957 [11]. The wave effects of this explosion were well studied. That is necessary to test the effectiveness of our model and the comparison with the parameters obtained from other models. Power of the explosion in Nevada was  $1.7 \text{ Kt}$  trotyl (equivalent  $M \approx 4$ ), that is corresponding to an energy  $E = 7.4 \cdot 10^{12} \text{ J}$ .

Now it should be defined the fundamental frequency, which is being main parameter in the formula (9). It is believed that in the case where the magnitude is known, it is possible not to use a spectral analysis and use the empirical relationship between the period and magnitude. However, in the range  $M = /3-5/$  such a relation is not defined. There is the equation that is considered fair for small magnitudes [12]

$$\lg T = 0.47M - 1.79 \quad (13)$$

According to this equation, the main (peak) frequency should be  $\approx 2.4 \text{ Hz}$  when  $M \approx 3$ . Obviously, for  $M \approx 4$ , it will be even less. In fact, for the explosion in the Nevada, main frequency was significantly greater, because fixed frequency were in the ranged  $/6-40/ \text{ Hz}$ . Consequently, the fundamental frequency of this interval should be considered as  $6 \text{ Hz}$ . However, it should be noted that during the arrival of the first head wave there were two peaks of frequency  $3 \text{ Hz}$  and  $7.5 \text{ Hz}$  at the same time. Then, after considerable delay, there were two peaks at  $10 \text{ Hz}$  and  $5 \text{ Hz}$ . This is not a very significant difference from the frequency of the first registered interval. This may have been caused by heterogeneity of the medium, as well as by the records error. The observations were made at a distance of about  $600 \text{ km}$  from the explosion site. Due to mention above appropriateness of the formula (13) seems doubtful.

Thus, the frequencies:  $\omega_2 = 6 \omega_3 = 12 \text{ Hz}$  were used to determine the characteristic radii of the Nevada explosion from formula (9). At the same time, the most probable value of the velocity

should be used, which, according to our model, can be assumed equal the velocity rate of body waves  $6.1 \text{ km/s}$  [11]. Consequently, since  $V_m \approx 0.8V_p$ , we will have  $V_p = 7.6 \text{ km/s}$ . As a result, the specific values have been obtained for fundamental and first harmonic from the two equations (9) characterizing the area of underground nuclear explosions  $r = 1.84 \text{ km}$  (the radius of the surface of the generation of body waves) and  $R = 1 \text{ km}$  (radius of the seismic source).

For comparison the well-known empirical formula for seismic source radius (in kilometers) and the magnitude can be used [13]

$$\lg R = -1.67 + 0.42M \quad (14)$$

According to this formula  $R \approx 1 \text{ km}$ , that is identical to the value obtained by our model for  $M \approx 4$ . Using  $R$  we can determine the volume of the seismic source  $V_c = \frac{4}{3}\pi R^3$  and seismic energy  $E_c = eV_c$ , where  $e$  is the density of the elastic strain energy. According to [5,6,12], this parameter is  $e \approx 10^{-4} \text{ J}$ . Seismic energy is approximately 5-8% of the total energy of the explosion, so its value will be  $(5 \div 8,4) \cdot 10^{12} \text{ J}$ , which correspond to the nuclear explosions with power in the range of  $2-1.2 \text{ kt}$  trotyl. Thus, in this case a good agreement have been established between the known value and corresponding interval of the modelled value of the total energy of an underground nuclear explosion in by the formula (9). Obviously, it is necessary to evaluate the quantitative effect of classical mathematical scheme's modification, which is the basis of our model. For this should be compared the result obtained for the hollow body with the result corresponding to a continuous area for the same value of the fundamental frequency. In this case, in the formula (9) must be regarded as  $R = 0$ , the following expression will be obtained from which we can determine the radius of the body, approximating the area of a point explosion

$$\omega_n^2 = \frac{\alpha V_m^2}{r^2} [(n-1)(n+2)n] \quad (15)$$

From the equation (15) for  $\omega_2 = 6 \text{ Hz}$  we obtain:  $r \approx 1,95 \text{ km}$ . If this magnitude is considered as the radius of the source, then the total energy of a point explosion will have a range that is much greater than the energy of the explosion in Nevada:  $E = (3.5 \div 5.8) \cdot 10^{13} \text{ J}$ . Thus, if this range is compared with a range derived above for the hollow body, the efficiency of our model giving real value of the energy is quite apparent.

For comparative analysis, the main frequency can be identified, which is given by equation (12) and formula (10). For this we use the radius of the area of the point explosion:  $r \approx 1,85 \text{ km}$ , because according to the formula (9) it corresponding to the frequency  $\omega_2 = 6 \text{ Hz}$ . In particular, the first root of the numerical solution transcendental equation (12) is  $kr = 0.5$ . Since  $\omega_k = V_p k$ , for  $k = 1/r$ , and  $V_p \approx 7.6 \text{ km/s}$ , we obtain:  $\omega_k \approx 2 \text{ Hz}$ . Further, in formula (10) is presented the velocity of shear wave. Its value when  $V_p \approx 7.6 \text{ km/s}$ , is equal to:  $V_s \approx 0.6V_p \approx 4.6 \text{ km/s}$ . Therefore, according to this model, the fundamental frequency  $\omega_0 = 5 \text{ Hz}$ . Based on these estimates, it can be concluded that in most cases there are small difference between the results of the formula (9), equation (12) and (10). But it is apparent disagreement with the empirical relation (13). These

facts serve as a clear demonstration of the effectiveness of our own model for mechanical vibrations of the point explosion. In particular, it is the only one using only the velocity of seismic body waves, as well as their fundamental frequency and the first harmonic explicitly defines the volume of the point explosion and its total energy.

For clarity of the formula (9), figure 1 illustrates the dependence of the radii  $r$  and  $R$  on the fundamental frequency  $\omega_2$  when the ratio  $\frac{\omega_3}{\omega_2} \approx 2$ . Here a seismic body wave velocity is  $V_m \approx 6.1 \text{ km/s}$ .

It should be noted that the assumption of the harmonic nature of the seismic waves is quite rude. This requirement becomes very hard when data are recorded on a great distance from the explosion or the weak earthquake. Obviously, in the real environment, due to its heterogeneity, the ratio between the fundamental frequency and the first harmonic will change. During the solution of the inverse problem, this ratio should be determined only by the harmonic analysis of the data.

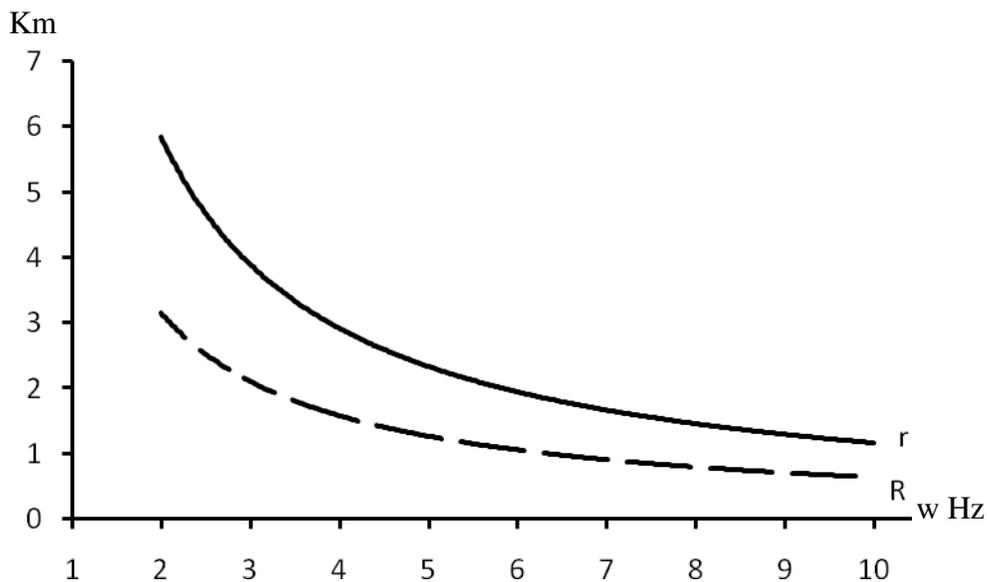


Figure 1. The dependence of the radii  $r$  (solid line) and  $R$  (dash line) on the fundamental frequency

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## **wertilovani afeTqebis seismuri energiis modelireba moculobiTi talRebis sixSiruli speqtris saSualebiT**

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reziume

amoxsnilia Sebrunebuli amocana, romlis mizans warmoadgens im moculobiTi seismuri talRebis diskretuli sixSiruli speqtris modelireba, romelTa generaciis mizezs warmoadgens xelovnuri wertilovani afeTqeba an susti bunebrivi miwisZvra ( $M$ ). gamoyenebulia siTxis wveTis sakuTari hidromeqanikuri rxvebis modelirebis cnobili analizuri meTodi da SemoTavazebulia sicarielis mqone wertilovani afeTqebis aris sferuli modeli. naSromis siaxles warmoadgens eileris radialuri gantolebis sruli amonaxsnis gamoyeneba. klasikuri sqemis, romelSic gamoiyeneboda mxolod Sinagani amonaxsni, aseTi modifikacia maTematikurad sakmaod koreqtulia, radganac gulisxmobs seismuri keris rxewis virtualobas. amrigad, moculobiTi seismuri talRebis sixSireTa diskretuli speqtris saSualebiT, SesaZlebeli xdeba sicarielis mqone sferuli sxულიT aproqsimirebuli wertilovani afeTqebis (susti miwisZvris) parametrebis gansazRvra. aseTi maxasiaTeblebia: seismuri keris (plastikurobis zona) radiusi da wrfivi drekadobis aris radiusi. amis Semdeg SesaZlebelia wertilovani afeTqebis an susti miwisZvris seismuri da sruli energiis sididis sakmaod martivad Sefaseba.

## **Моделирование сейсмической энергии точечного взрыва при помощи спектра частот объемных волн**

### З. Кереселидзе, Н. Церетели

#### Резюме

Решается обратная задача, цель которой заключается в моделировании дискретного спектра частот объемных сейсмических волн, генерированных искусственным точечным взрывом или слабым естественным землетрясением ( $M < 2$ ). Предлагается сферическая модель полый области точечного взрыва и используется известный аналитический метод для моделирования собственных гидромеханических колебаний жидкой капли. Новшеством, примененным в работе, является использование полного решения радиального уравнения Эйлера. Такая модификация классической схемы, в которой применялось лишь внутреннее решение, является математически достаточно корректной, т.к. подразумевает виртуальность упругих колебаний сейсмического очага. В результате, при помощи дискретного спектра частот объемных сейсмических волн, можно определить линейные параметры области точечного взрыва (слабого землетрясения), аппроксимируемой полым телом сферической формы. Такими линейными характеристиками являются радиус сейсмического очага (зона пластичности) и радиус зоны линейной упругости. После этого можно достаточно просто оценить сейсмическую и полную энергию точечного взрыва либо слабого землетрясения.