The Journal of the Georgian Geophysical Society,
Issue (B), Physics of Atmosphere, Ocean, and Space Plasma, v. 17, 2014, pp. 70-76

Linear generation of Khantadze waves by moedified Rossby waves in ionospheric shear flows

R. G. Chanishvili[1,2], O. A. Kharshiladze[1,3] and E. S. Uchava[1,2,3]

[1] M. Nodia Institute of Geophysics, Tbilisi State University, Tbilisi, Georgia

[2] Abastumani Astrophysical Observatory, Ilia State University, Tbilisi, Georgia

[3] Faculty of Exact and Natural Sciences, Tbilisi State University, Tbilisi, Georgia

Abstract

We study ionospheric zonal shear flow non-normality induced linear coupling of planetary scale modified Rossby waves and Khantadze waves on the basis of nonmodal approach. We demonstrate that the modified Rossby waves generate Khantadze waves due to the coupling for a quite wide range of ionospheric and shear flow parameters.

1. Introduction

The main ingredients of the ionosphere's planetary scale activity are spatially inhomogeneous zonal winds (shear flows) and slow and fast waves. The slow waves are Rossby waves modified by the geomagnetic field. The fast ones caused by the Hall effect, have significant magnetic fluctuations and called Khantadze waves [1,2]. The shear flow can drastically affect the energy and structure of the slow and fast waves. For instance, the flow forms nonlinear structures of waves [3]. The modal/spectral linear approach specifies the grows of slow and fast wave harmonics at nonzero second derivative of the basic/shear flow $U_0^{''}=0$. However, in the modal analysis, the focus is on the asymptotic stability of flows and finite time period dynamics (so-called, the transient dynamics) left for speculation.

In the 1990s, the emphasis shifted from the analysis of long-time asymptotic flow stability to the study of transient behavior on the basis of so-called non-modal approach. (The non-modal analysis involves the change of independent variables from the laboratory to a moving frame and the study of the temporal evolution of spatial Fourier harmonics (SFH) of perturbations without any spectral expansion in time.) This fact resulted a breakthrough of the hydrodynamic community in the analysis of the linear dynamics of smooth shear flows (e.g. see.[4-7]). According to the non-modal approach, the early transient period for the perturbations reveal rich and complicate behavior in smooth (without inflection point) shear flows: the linear dynamics of perturbations in the flows are accompanied by intense temporal energy exchange processes between the background flow and perturbations and between different modes of perturbations.

The purpose of the present paper is the demonstration of the linear generation of fast/Khantadze waves by modified Rossby waves in ionospheric zonal shear flows. The paper is organized as follows. In Sect. 2 we present the physical model and dynamical equations in the spectral plane. In Sect. 3 we present a numerical analysis of the dynamical equations and the summary.

2. Physical model and equations

As is commonly done, we introduce a local Cartesian coordinate system that rotates with the planet (with angular velocity Ω_0 and is centered on a latitude θ_0 and a distance R_0 from the planet center (on the ionospheric E-layer in our case). The x-axis is directed to the east, the y-axis to the north, and the zaxis in the local vertical direction. We study the linear dynamics of planetary scale perturbations in the conductive ionospheric E-layer with account of the latitudinal inhomogeneity (over θ that the same, over the coordinate y) of the Coriolis parameter f and the geomagnetic field B_{0z} :

$$f(\theta) = 2\Omega_{0z}(\theta) = 2\Omega_0 \sin\theta \approx f_0 + \beta \cdot y, \tag{1}$$

with $f_0 = 2\Omega_0 \sin\theta_0$

and

$$\beta = \left(\frac{\partial f}{\partial y}\right)_{\substack{R=R_0 \\ \theta=\theta_0}} = \frac{2\Omega_0 \cos\theta_0}{R_0},$$

$$H_{0z}(\theta) \approx H_{0z}(\theta_0) - \beta_H \cdot y, \tag{2}$$

with

$$\beta_H = -\left(\frac{\partial H_{0z}}{\partial y}\right)_{\substack{R=R_0 \\ \theta=\theta_0}} = \frac{H_p \cos\theta_0}{R_0},$$

H_p is the geomagnetic field at the pole. The zonal flow (directed along x axis) has latitudinal shear $U_0 = (Sy, 0, 0)$.

As noted in the introduction, we base our study on the simplified set of 2D equations (see Eqs.(3) of [3]) written in the linear limit. The set takes into account Hall's effect and facts that planetary scale motions do not perturb density and concentration of the medium components [12]. So, the starting equations for our analysis are:

$$\left(\frac{\partial}{\partial t} + Sy\frac{\partial}{\partial x}\right)\psi + \beta\frac{\partial \psi}{\partial x} - \frac{1}{4\pi\rho_0}\beta_H \frac{\partial h_z}{\partial x} = 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t}+Sy\frac{\partial}{\partial x}\right)h_z-\frac{\alpha}{4\pi}\beta_H\frac{\partial h_z}{\partial x}+\beta_H\frac{\partial\psi}{\partial x}=0 \tag{4}$$

where, $\psi(x,y,t)$ is the stream function of the neutral-gas perturb motion in the horizontal plane; $h_z(x,y,t)$ - the vertical z-component of magnetic field strength perturbation; $\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2$ - the two dimensional Laplacian; $\alpha=c/eN$, - the Hall parameter; c - speed of light; e - value of the electron charge; N - the concentration of electrons.

Let's introduce non-dimensional variables and parameters:

$$t\Omega_0\cos\theta_0\Rightarrow t,\quad \frac{(x,y)}{R_0}\Rightarrow(x,y),\quad \frac{S}{\Omega_0\cos\theta_0}\Rightarrow S,$$
$$\frac{\rho_0\Omega_0}{H_p}\alpha\Rightarrow\alpha,\quad \frac{\psi}{\Omega_0R_0^2}\Rightarrow\psi,\quad \frac{h_z}{H_p}\Rightarrow h_z, \tag{5}$$

and rewrite the dynamical system in the non-dimensional form:

$$\left(\frac{\partial}{\partial t}+Sy\frac{\partial}{\partial x}\right)\psi+2\frac{\partial\psi}{\partial x}-V_A^2\frac{\partial h_z}{\partial x}=0, \tag{6}$$

$$\left(\frac{\partial}{\partial t}+Sy\frac{\partial}{\partial x}\right)h_z-\alpha V_A^2\frac{\partial h_z}{\partial x}+\frac{\partial\psi}{\partial x}=0, \tag{7}$$

Where $V_A=H_p/(\sqrt{4\pi\rho_0}\Omega_0R_0)$ defines the neutral-gas-loaded Alfven velocity normalised on Ω_0R_0 .

The form of the dynamical equations permit a decomposition of perturbed quantities into shearing plane waves (so-called, Kelvin waves). In fact, these waves represent spatial Fourier harmonics (SFHs) with time-dependent amplitudes and phas es (e.g., see [8-10]):

$$F(x,y,t)=\hat{F}(k_x,k_y(t),t)\exp(ik_xx+ik_y(t)y), \tag{8}$$

$$k_y(t)=k_{y0}-Sk_xt, \tag{9}$$

where $F=\{\psi,h_z\}$ denotes the perturbed quantities and $\hat{F}=\{\hat{\psi},\hat{h}_z\}$ - the amplitudes of the corresponding SFHs.

Substituting Eq.(8) into Eqs.(6,7) and introducing $\tilde{\phi}=\hat{\psi}\cdot k^2(t)$, $k^2(t)=k_x^2+k_y^2(t)$, one can get the following system:

$$\frac{\partial\tilde{\phi}}{\partial t}=2\frac{ik_x}{k^2(t)}\tilde{\phi}-ik_xV_A^2\hat{h}_z,\qquad \frac{\partial\hat{h}_z}{\partial t}=-\frac{ik_x}{k^2(t)}\tilde{\phi}+ik_x\alpha V_A^2\hat{h}_z, \tag{10}$$

In the shearless limit, $S=0$, k_y and k^2 are time independent and coefficients of the dynamical equations (10) are constant. Hence, one can use the Fourier expansion of $\tilde{\phi}$ and $\hat{h}_z$ in time,

$\sim \exp(-i\omega t)$, and obtain the dispersion equation of the considered system in the shearless limit:

$$k^2\omega^2 + (\alpha k^2 V_A^2 + 2)k_x\omega + (2\alpha - 1)k_x^2 V_A^2 = 0, \tag{11}$$

Solutions of the dispersive equation are

$$\omega_{f,s} = -\frac{k_x}{2k^2}(\alpha k^2 V_A^2 + 2 \pm \sqrt{(\alpha k^2 V_A^2 - 2)^2 + 4k^2 V_A^2}). \tag{12}$$

Eqs.(12) describe fast and slow wave harmonics. At $\alpha k^2 V_A^2 >> 1$ one can write:

$$\omega_s = -\frac{k_x}{k^2}\frac{2\alpha - 1}{\alpha}, \qquad \omega_f = -\alpha k_x V_A^2, \tag{13}$$

where ω_s is the frequency of the modified/magnetized Rossby waves. This slow wave mode propagates either eastward or westward depending on the sign of $2\alpha - 1$; ω_f is the frequancy of the Khantadze waves [3,11]. This wave mode has a substantial magnetic component and propagates rapidly westward.

One can also easily get the expression of the spectral energy of perturbations:

$$E = E_k + E_m = \frac{|\tilde{\phi}|^2}{k^2(t)} + V_A^2 |\tilde{h}_z|^2 . \tag{14}$$

The spectral energy is the sum of quadratic forms of stream function and magnetic field harmonics, i.e., the sum of spectral kinetic and magnetic energies. The magnetic energy is mostly connected with the Khantadze waves.

3. Numerical results and summary

For numerical integration of Eqs. (10) we use a standard Runge–Kutta scheme (MATLAB ode34 RK implementation). In order to study generation of Khantadze waves by modified Rossby waves, we initially imposing in Eqs. (10) a tightly leading ($k_x = 1,\ k_y(0) = 10$, i.e., $k_y(0)/k_x >> 1$) pure magnetised Rossby wave harmonic:

$$\tilde{\phi}(0) = 1, \qquad \tilde{h}_z(0) = \frac{1}{k_x V_A^2}\left(\omega_s(0) + \frac{2k_x}{k^2(0)}\right) = \frac{1}{\alpha k^2(0) V_A^2}. \tag{15}$$

For the ionospheric E-layer parameters [11]: electrons concentration - $N = 10^5 cm^{-3}$; neutral gas concentration - $N = 10^{13} cm^{-3}$; Neutral gas mass density - $\rho_0 = 4.175 \cdot 10^{-10} gr \cdot cm^{-3}$. These parameters give $V_A^2 = 0.022$, $\alpha = 38$. For these parameters $\tilde{h}_z(0) = 1.2 \cdot 10^{-2}$. Finally, for these initial conditions and nondimensional shear parameter $S = 0.1$ Eqs. (10) gives the time dynamics of expansion of $\tilde{\phi}$, $\tilde{h}_z$, $E_k(t)$ and $E_m(t)$ presented on Figs 1-4. The figures show that the magnetic field is negligibly perturbed in the leading phase – we have mostly stream function (kinematic) perturbations. However, at $k_y(t)/k_x < 1$ the linear coupling starts the generation of magnetic field oscillations (see Fig2) that relates to the Khantadze waves. Figs. 3 and 4 indicate that, in the initial trailing region ($0 > k_y(t)/k_x > -1$), the spectral magnetic energy becomes comparable to the spectral kinetic one, while, in the tightly trailing region ($-1 >> k_y(t)/k_x$), the spectral magnetic energy highly exceeds the spectral kinetic one. So, starting with a tightly leading modified Rossby waves, finally, the linear dynamics give the related harmonic of tightly trailing magnetic oscillations – the Khantadze waves. The spectral energy of the generated Khantadze waves more than an order of magnitude higher than the spectral energy of the initial tightly leading modified Rossby waves for the considered ionospheric and shear flow parameters.

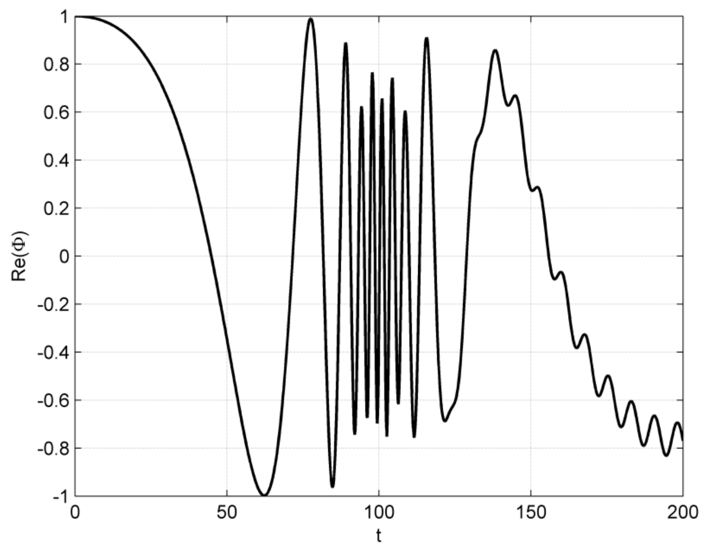

Fig. 1. The evolution of the real part of a stream function harmonic, $\mathrm{Re}\,\tilde{\phi}$, for initial conditions correspond to the pure magnetised Rossby wave harmonic and parameters

$k_x = 1,\ k_y(0) = 10,\ V_A^2 = 0.022,\ \alpha = 38.$

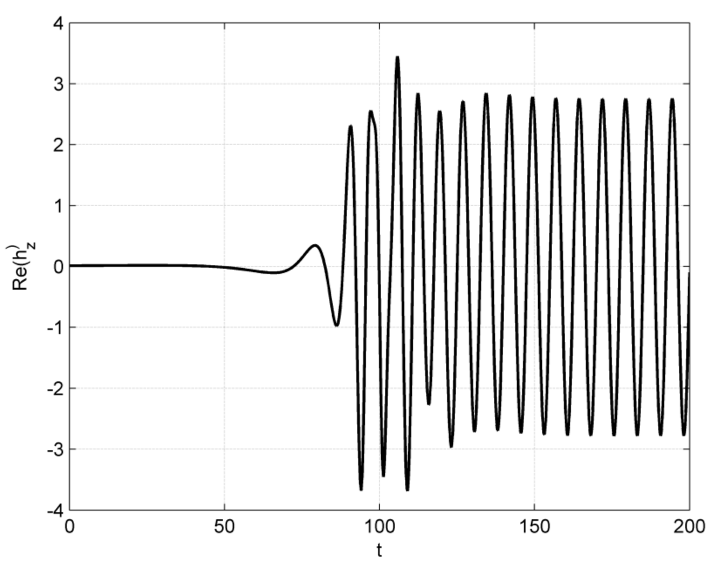

Fig. 2. The evolution of the real part of a magnetic field strength harmonic, $\mathrm{Re}\,\tilde{h}_z$, for the same case as in Fig. 1.

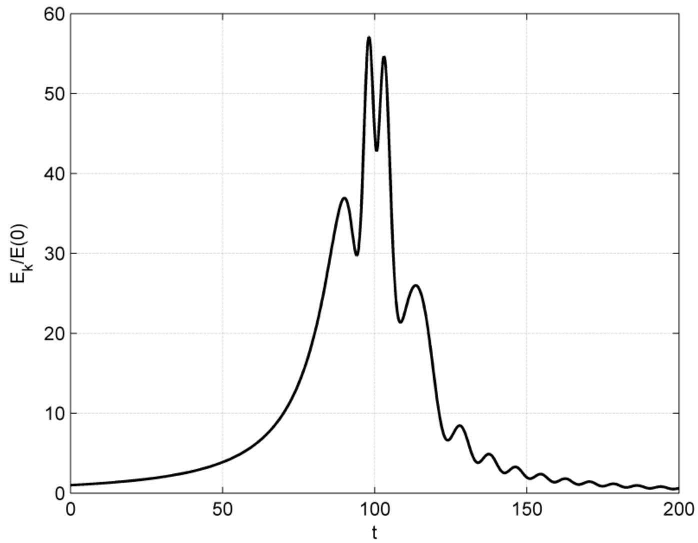

Figure 3. Normalized spectral kinetic energy vs. time, $E_k(t)$, for the same case as in Fig. 1.

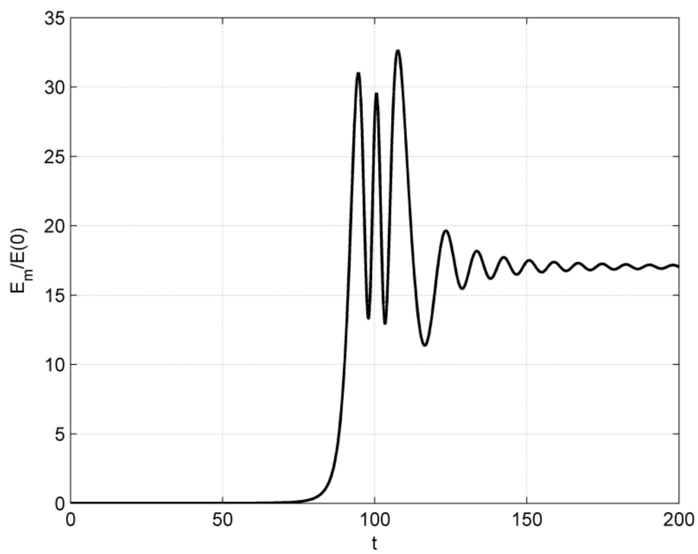

Figure 4. Normalized spectral magnetic energy vs. time, $E_m(t)$, for the same case as in Fig. 1.

Acknowledgments: This work was supported in part by GNSF grant 31/14 and CRDF/GRDF grant A60765.

References

[1] Khantadze, A. G., Bull. Acad. Sci. Georgian SSR, 123(1), 69–71,1986 (in Russian).
[2] Khantadze, A. G., J. Georgian Geophys. Soc., 4B, 125–127,1999 (in Russian).
[3] T. D. Kaladze, L. Z.Kahlon, W. Horton, O. Pokhotelov and O. Onishchenko, EPL (Europhysics Letters),2014, V.106, 29001 doi: 10.1209/0295-5075/106/29001
[4] Gustavsson L. H., 1991, J. Fluid Mech., 224, 241
[5] Reddy S. C., Schmid P. J., Henningson D. S., 1993, SIAM J. Appl. Math., 53, 15
[6] Farrell B. F., Ioannou P. J., 1993, Phys. Fluids A, 5, 1390
[7] Henningson D. S., Reddy S. C., 1994, Phys. Fluids, 6, 1396
[8] Chagelishvili G.D., Chanishvili R.G. and Lominadze J.G., JETP, 2002, **94,** 434
[9] Mamatsashvili G.R., Chagelishvili G.D, Monthly Notices of the Royal Astronomical Society, 2007, **381**, 809
[10] Tevzadze A. G., Chagelishvili G. D., Bodo G. And Rossi P., Monthly Notices of the Royal Astronomical Society, 2010, **401**, 901
[11] Kaladze T., Horton W., Kahlon L., Pokhotelov O., Onishchenko O. Journal of Geophysical Research: Space Physics V. 118, Issue 12, pages 7822–7833, 2013
[12] Gossard E., Hook W. Waves in Atmosphere. Elsevier, 1975

ხანთაძის ტალღების წრფივი გენერაცია მოდიფიცირებული როსბის ტალღებით იონოსფერულ წანაცვლებით დინებებში

რ. ჭანიშვილი, ო. ხარშილაძე, ე . უჩავა

იონოსფერულ ზონალურ წანაცვლებით დინებების არაორთოგონალურობით განპირობებული პლანეტარული მასშტაბის მოდიფიცირებული როსბის ტალღებისა და ხანთაძის ტალღების წრფივი კავშირი შესწავლილია არამოდალური მიდგომის საფუძველზე. ნაჩვენებია, რომ ამ კავშირის შედეგად, მოდიფიცირებული როსბის ტალღები აგენერირებენ ხანთაძის ტალღებს იონოსფეროსა და წანაცვლებითი დინების პარამეტრების საკმაოდ ფართე დიაპაზონისათვის.

Линеиная генерация волн Хантадзе модифицированними волнами Россби

в ионосферных сдвиговых течениях

Р. Чанишвили, О. Харшиладзе, Е. Учава

Линейная связь между планетарных масштабов модифицированными волнами Россби и волнами Хантадзе обусловленная неортогональностью ионосферных сдвиговых течениий изучена на основе немодального подхода. Показано, что в результате этой связи генерируются волны Хантадзе для широкого диапазона параметров ионосферы и сдвигового течения.