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On the preparation of the initial data for prognostic problem of the baroclinic ocean dynamics

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Abstract

It is well known that quality of the hydrophysical fields received as a result of realization of the prognostic model of the dynamics of the baroclinic ocean considerably depends on quality of the input data. In the present study, on the basis of the conjugated equations and the perturbation theory the algorithm for specification of the observational data on the non-stationary processes, used in the boundary conditions on the free sea surface, is offered. With the purpose of convenience, at first the algorithm on preparation of the initial data for the prognostic model of the ocean dynamics is considered on an example of two-dimensional, xoz-coordinate plane, transfer-diffusion equation for a substance, and then - for a three-dimensional problem of dynamics of baroclinic ocean.

1. Introduction

At solution of problems of ocean dynamics, especially with taken into account non-stationary atmospheric processes, there is a number of problems. Among them the preparation of initial data which are absent not only for the World ocean but even for the internal seas, is rather important. However, this problem can be solved successfully by means of hydrodynamic methods. Considering a problem of forecasting of ocean currents, it is natural to assume that initial fields, especially in the upper layer, will be formed basically under forcing of atmospheric non-stationary conditions, first of all by wind and thermal modes at the free ocean surface. With the purpose of preparation of the initial data for the problem of the ocean dynamics, at first it is necessary to solve a problem about a climatic condition of the ocean with zero initial and climatic boundary conditions on a free surface of the ocean. Then, the received fields are used as the initial data in the problem of ocean dynamics, where the real data on weather conditions are used as boundary conditions on the ocean free surface. As a result of solution of this problem we find the solution for perturbation in oceanic circulation under concrete meteorological situations in the atmosphere within some time period. Thus, taking into account real perturbations of meteorological processes in the atmosphere, the constructed hydrophysical fields can be used as initial conditions at solution of the problem of forecast of ocean circulation [1].

As a whole the marine forecast can be divided into two stages. At the first stage, the problem allowing to receive the information about the stationary (climatic) condition of currents and fields of temperature, salinity, and density under influence wind stress, heat and salinity fluxes on the ocean free surface, is solving. At the second stage, the received climatic hydrophysical fields, when there are real continuous non-stationary meteorological fields in the atmosphere within the time period $0 \le t \le t_m$ previous to a prognostic interval $t_m \le t \le T$, are used as initial fields for solution of the prognostic model of oceanic processes.

It is necessary to note that the local meteorological information is very sensitive to unpredictable meteorological "noise" that is essentially reflected on results of mathematical model. At present, numerical prognostic models are developed, which qualitatively adequately describe the physical processes occurring in the oceans and seas [1-19]. However, for qualitatively true description of the forecast of hydrophysical fields it is necessary to use assimilation of various observations.

Assimilation of measurements represents the procedure, which allows to combine the observational data with modeling calculations for maximally adequate reproduction of a real state of the environment [20].

In the present study, on the basis of the conjugated equations and the perturbation theory the algorithm on specification of the given observations on the non-stationary processes used in the boundary conditions on the free sea surface sea is offered. As is known, quality of the hydrophysical fields received as a result of realization of the prognostic model of the dynamics of the baroclinic sea considerably depends on quality of these data.

With the purpose of convenience, at first the algorithm on the preparation of the initial data for a prognostic model is considered on an example of two-dimensional (xoz-vertical plane) transfer-diffusion equation for a substance, and then – for a three-dimensional problem of dynamics of baroclinic ocean.

The theory of conjugated equations and the theory of perturbations for a long-term weather forecast and protection of the environment have been developed in Marchuk's numerous articles and presented in detail in monographies [1, 2]. The method of preparation of initial data developed in this study is based on above mentioned researches.

2. Two-dimensional prognostic problem

Let us consider, for example, a 2D transfer-duffusion equation in the area Ω (vertical section in the coordinate system xoz, with z-axis directed vertically downward) with depth H. Thus, we have an equation

$$T_t + div\vec{u}T - \mu'_T T_{xx} - \nu'_T T_{zz} = 0 \qquad (1)$$

with boundary and initial conditions

$$\begin{aligned} \nu'_T T_z &= Q_T && \text{on} && z = 0, \\ T_z &= 0 && \text{on} && z = H, \qquad (2) \\ T_x &= 0 && \text{on} && x = 0, L, \\ T &= T_0(x, z) && \text{at} && t = 0. \qquad (3) \end{aligned}$$

Here T is the deviation of temperature of marine water from its standard values $\overline{T}(z)$, $Q_T = \beta(T_M - T_B) - R$, T_M and T_B are deviations of climatic temperature of the ocean surface and the air at the level z = 2 m, respectively; β is the factor of ocean heat transfer; R = S+A+B, where R is the radiation flux through the unit square in the plane xoy on the level z; S is the flux of short-wave solar radiation; A is the flux of long-wave radiation, directed downward and B is the flux of long-wave radiation, directed upward, T_0 is given function; L is the size of the solution domain along x; $\mu'_T = \mu_T + \delta\mu_T$ and $\nu'_T = \nu_T + \delta\nu_T$ are horizontal and vertical diffusion coefficients, respectively, where $\delta\mu_T$ and $\delta\nu_T$ are already defined on the basis of conjugated equations and the theory of small perturbations [14, 21, 22]; u and w are known functions, which are the components of flow velocity vector $\vec{u}$, and satisfy the continuity equation

$$div\vec{u} = 0$$

and boundary conditions

$$\begin{aligned} u &= 0 && \text{on} && x = 0, L, \\ w &= 0 && \text{on} && z = 0, H. \end{aligned}$$

Now, with the purpose of creation of a conjugate problem let us multiply the equation (1) by some function T^* and integrate the result on time variable from 0 to some t_m and on the area **Ω**. Then, we will receive

$$\int_0^{t_m}\iint_\Omega T^*\left(T_t + div\vec{u}T - \mu_T' T_{xx} - \nu_T' T_{zz}\right)d\Omega dt = 0 . \qquad (4)$$

The left part of the equality (4) we will transform so that behind brackets under integral there was function T, and in the brackets – the differential parity containing function T^*. With this purpose we consider the operator

$$\Lambda' = \Lambda_1 + \Lambda_2' ,$$

where

$$\Lambda_1 T = div\vec{u}T , \qquad \Lambda_2' T = -\mu_T' T_{xx} - \nu_T' T_{zz} .$$

Let

$$\left(g,h\right) = \iint_\Omega ghd\omega ,$$

where integration is made in the area of definition of some functions g and h. Then, with the help of the Lagrangian identity and homogeneous boundary conditions, corresponding to conditions (2) and similar conditions for function T^*, it is evident that

$$\left(T^*,\Lambda' T\right) = \left(T^*,\Lambda_1 T\right) + \left(T^*,\Lambda_2' T\right) = \left(\Lambda_1^* T^*,T\right) + \left(\Lambda'^*{}_2 T^*,T\right) =$$
$$= \iint_\Omega T\left(-div\vec{u}T^* - \mu_T' T^*_{xx} - \nu_T' T^*_{zz}\right)d\Omega .$$

Thus, we have

$$\Lambda_1^* = -\ \Lambda_1 \quad \text{и} \quad \Lambda_2'^* = \Lambda_2' .$$

Let f^* and $T^*_{t_m}$ are known functions of coordinates, which we will define later. If we assume that the function T^* satisfies the conditions

$$\nu_T' T_z^* = \beta T_M^* + f^* \qquad \text{on} \qquad z = 0,$$
$$T_z^* = 0 \qquad \text{on} \qquad z = H, \qquad (5)$$
$$T_x^* = 0 \qquad \text{on} \qquad x = 0, L,$$
$$T^* = T^*_{t_m} \qquad \text{on} \qquad t = t_m , \qquad (6)$$

after corresponding transformations and using (2)–(3) and (5)–(6) from (4) we receive

$$\iint_\Omega \left(T^*_{t_m} T_{t_m} - T_0^* T_0\right)d\Omega + \int_0^{t_m}\iint_\Omega T\left(-T_t^* - div\vec{u}T^* - \mu_T' T^*_{xx} - \nu_T' T^*_{zz}\right)d\Omega dt =$$
$$= \int_0^{t_m}\int_0^L T^*_{z=0}\left(\beta T_B + R\right)dxdt + \int_0^{t_m}\int_0^L T_M f^* dxdt. \qquad (7)$$

Assume that T^* satisfies the equation

$$- T_t^* - div\vec{u}T^* = \mu'_T T_{xx}^* + \nu'_T T_{zz}^* \tag{8}$$

at boundary and initial conditions (5)-(6). The equation (8) is conjugate with respect to the equation (1).

Let us multiply the equation (1) by T^*, the equation (8) – by T, and integrate them on the area Ω and on time in limits from 0 to t_m, the result we will subtract from each other. Then with using the boundary conditions (2)-(3), (5)-(6), after corresponding transformations, we will receive the quality

$$\iint_{\Omega} \left(T_{t_m}^* T_{t_m} - T_0^* T_0 \right) d\Omega - \int_0^{t_m}\int_0^L T_{z=0}^* \left(\beta T_B + R \right) dxdt = \int_0^{t_m}\int_0^L T_{z=0} f^* dxdt \, . \tag{9}$$

Let us assume that

$$f^* = \frac{\nu_T'^2 \gamma_T}{g} \delta(t - t_m, x - x_0) = \begin{cases} \dfrac{\nu_T'^2 \gamma_T}{g}, & if \quad t = t_m, x = x_0 \\ 0, & if \quad t \neq t_m, x \neq x_0 \, . \end{cases}$$

Then, as

$$\int_0^{t_m}\int_0^L T_{z=0} f^* dxdt = \frac{\nu_T'^2 \gamma_T}{g} T_{z=0,t_m}(x_0),$$

and if we assume that $T_{t_m}^* = 0$ and $T_{M_{t_m}}(x) \equiv T_{z=0,t_m}(x)$, from (9) we have

$$T_{M_{t_m}}(x_0) = -r_T \left(\iint_{\Omega} T_0^* T_0 d\Omega + \int_0^{t_m}\int_0^L T_{z=0}^* \left(\beta T_B + R \right) dxdt \right), \qquad r_T = \frac{g}{\nu_T'^2 \gamma_T}. \tag{10}$$

This formula specifies a relation between temperature in the given point on the surface z = 0 of the area Ω at the time moment t_m, initial relation (at t = 0) of the ocean and boundary conditions from (2). In the formula (10) T_0 is given at the initial time moment, but T_0^* is the solution of the problem (8), (5)–(6) at condition $T_{t_m}^* = 0$.

Thus, to use the formula (10) it is necessary for each fixed point x_0 to solve the conjugate problem (8), (5)-(6). This circumstance specifies that to use the formula (10) is not effectively, especially, when we consider a three-dimensional problem of the baroclinic ocean dynamics. However, to simplify this problem, the surface σ of the area Ω is divided, for example, on two parts σ_1 and σ_2 and the average anomaly of temperature in each of them is defined. With this purpose assume in (7) that

$$f^* = \begin{cases} \dfrac{\nu_T'^2 \gamma_T}{g\sigma_i} \delta(t - t_m), & if \ x \in \sigma_i \qquad (i = 1,2) \\ 0, & out \ of \ the \ given \ domain \end{cases}$$

where
$$\delta(t - t_m) = \begin{cases} 1, & if \quad t = t_m \\ 0, & if \quad t \neq t_m \end{cases}.$$

Our goal is an improvement of the quality of the initial data at $t = t_m$ in the prognostic model, but as is known, it essentially depends on quality of the field of temperature anomaly given on the boundary conditions on the ocean free surface. In this connection, following the results received in [1], we construct corresponding functional for calculation average anomaly of temperature $\delta\left(\overline{T}_{M_{t_m}}\right)$ on the ocean free surface at the time moment t_m and then we define the perturbing state (anomaly) of temperature on equality $T'^{\sigma_i}_{M_{t_m}} = T_{M_{t_m}} + \delta\left(\overline{T}^{\sigma_i}_{M_{t_m}}\right)$, where $T^{\sigma_i}_{M_{t_m}}$ is climatic value of temperature on z = 0 at the time moment t_m. Function $T'^{\sigma_i}_{M_{t_m}}$ defined by this way is used on the boundary conditions on z = 0 for perturbed prognostic equation (the real state of the ocean we name perturbed, and the climatic state we assume as the basic "undisturbed" state of the ocean [1]).

Now we will designate average anomaly of temperature accordingly for subareas σ_i (i = 1, 2) as follows

$$\frac{1}{\sigma_i}\int_0^L T_{M_{t_m}} dx = \overline{T}^{\sigma_i}_{M_{t_m}} .$$

Then, we have

$$\int_0^{t_m}\int_0^L T_M f^* dxdt = \frac{\nu_T'^2 \gamma_T}{g\sigma_i}\int_0^L T_{M_{t_m}} dx = \frac{\nu_T'^2 \gamma_T}{g}\overline{T}^{\sigma_i}_{M_{t_m}} . \tag{11}$$

Considering (11) from (9), under the condition $T^*_{t_m} = 0$, accordingly for σ_1 and σ_2 we receive

$$\overline{T}^{\sigma_i}_{M_{t_m}} = -r_T\left(\iint_\Omega T_0^* T_0 d\Omega + \int_0^{t_m}\int_0^L T^*_{z=0}\left(\beta T_B + R\right)dxdt\right). \qquad \text{(i = 1, 2).} \tag{12}$$

Expressions (12) mean that average anomaly of temperature on σ_i is calculated by data on the interval $0 \le t \le t_m$. It is necessary to notice that the formulas received from (12) at i = 1, 2 are visually similar, though they differ by solution of the conjugate problems. Thus, the problem on definition of average anomaly of temperature on σ_i was reduced to the solution of the conjugate problem (8), (5) – (6) under the condition $T^*_{t_m} = 0$.

Expressions (12) are needed at consideration of the perturbation theory. With this purpose we will consider the perturbed equation

$$T_t' + div\vec{u}T' = \mu_T' T_{xx}' + \nu_T' T_{zz}' . \tag{13}$$

At the following boundary and initial conditions

$$\begin{aligned} &\nu_T' T_z' = Q_T' && \text{on} \quad z = 0, \\ &T_z' = 0 && \text{on} \quad z = H, \qquad (14)\\ &T_x' = 0 && \text{on} \quad x = 0, L, \\ &T' = T_0' && \text{on} \quad t = 0, \qquad (15) \end{aligned}$$

where T_0' is climatic value (the solution of the problem (1)–(3) at condition $T_0 = 0$),

$$Q_T' = \beta\left(T_M' - T_B'\right) - R\,, T_M' = T_M + \delta T_M\,, T_B' = T_B + \delta T_B\,.$$

Now we multiply the equation (13) by the conjugated function T^*, corresponding to the not perturbed (climatic) problem (1) – (3), and the conjugated problem (8) – by T', Then, results of these operations we will subtract from each other integrate both on time from 0 to t_m and on the area Ω. Similar to (12) we receive following expressions

$$\overline{T'}^{\sigma_i}_{M_{t_m}} = -r_T\left(\iint_\Omega T_0^* T_0' d\Omega + \int_0^{t_m}\int_0^L T_{z=0}^*\left(\beta T_B' + R\right)dxdt\right),\ (\text{i} = 1, 2). \tag{16}$$

Considering that

$$\overline{T'}^{\sigma_i}_{M_{t_m}} = \overline{T}^{\sigma_i}_{M_{t_m}} + \delta\left(\overline{T}^{\sigma_i}_{M_{t_m}}\right),\quad T_0'^{\sigma_i} = T_0^{\sigma_i} + \delta T_0^{\sigma_i},\ T_B'^{\sigma_i} = T_B^{\sigma_i} + \delta T_B^{\sigma_i},\ (\text{i} = 1, 2)$$

and subtracting from (16) equality (12), we come to the functionals for definition of values $\delta\left(\overline{T}^{\sigma_i}_{M_{t_m}}\right)$, (i = 1, 2) accordingly for subareas σ_i at the moment t_m. Thus, we have

$$\delta\left(\overline{T}^{\sigma_i}_{M_{t_M}}\right) = -r_T\left(\iint_\Omega T_0^* \delta T_0^{\sigma_i} d\Omega + \int_0^{t_m}\int_0^L \beta T_{z=0}^* \delta T_B^{\sigma_i} dxdt\right),\qquad (\text{i} = 1, 2).$$

Further, on equalities

$$T'^{\sigma_i}_{M_{t_m}} = T^{\sigma_i}_{M_{t_m}} + \delta\left(\overline{T}^{\sigma_i}_{M_{t_m}}\right)\qquad (\text{i} = 1, 2) \tag{17}$$

we define perturbed values of temperature anomalies on σ_i at the time moment t_m.
Now, let us consider the perturbed equation again

$$T_t' + div\vec{u}T' = \left(\mu_T + \delta\mu_T\right)T_{xx}' + \left(\nu_T + \delta\nu_T\right)T_{zz}' \tag{18}$$

at boundary and initial conditions (14)–(15), in addition, on the boundary conditions on z = 0 functions $T'^{\sigma_i}_{M_{t_m}}$, i = 1, 2, defined from formulas (17), are used.

Thus, at solution of the problem (18), (14), and (15) it is supposed that the heat flux on the ocean surface Q_T' is known at any moments of time previous prognostic time interval. Choosing as the initial condition at t = 0 climatic condition of ocean, we will adapt step by step the ocean for the real perturbations arriving from its surface. It is necessary to notice that in the perturbation problem (18), (14), and (15), except the specified values of anomalies of temperature $T'^{\sigma_i}_{M_{t_m}}$ (i = 1, 2), included in the boundary conditions on the surface of considered area Ω, are also used the perturbed values of factors turbulent diffusion $\mu_T' = \mu_T + \delta\mu_T$ and $\nu_T' = \nu_T + \delta\nu_T$ that considerably raises adequacy of the results received on the perturbation problem, with an existing condition of a thermal condition of ocean [22]. The received solution of the perturbed problem (18), (14), and (15) or ditto the information on a thermal mode of the ocean can be used as the initial data by consideration of prognostic model in the range of time $t_m \le t \le T$.

On the basis of the above-stated it is possible to summarize that the sequence of realization of the algorithm of preparation of the input information at the initial time moment $t = t_m$ for the prognostic problem is following:

1. On the basis of the theories of the conjugate equations and small perturbations the eddy factors $\delta\nu_T$ and $\delta\mu_T$ are defined and we can find $\mu_T' = \mu_T + \delta\mu_T$ and $\nu_T' = \nu_T + \delta\nu_T$;

2. The problem

$$\frac{\partial \widetilde{T}}{\partial t} + \Lambda' \widetilde{T} = 0,$$
$$\widetilde{T} = 0 \quad at \;\; t = 0,$$

is solving and there is supposed that the input data used on boundary conditions on the free surface are climatic data. The problem is solving before quasisteady state achievement.

3. In the range of time $0 \le t \le t_m$ the problem

$$\frac{\partial T}{\partial t} + \Lambda' T = 0,$$
$$T = \widetilde{T}_{t_{m_1}} \quad at \;\; t = 0$$

is solving. Here the input data used on the boundary conditions on the free surface are climatic data.

4. within the interval $0 \le t \le t_m$ at $T^*_{t_m} = 0$ the conjugate problem (8), (5), (6) is solving twice with taken into account σ_1 and σ_2 respectively.

5. It is supposed, that during any moments of time, previous to a predicted interval of time it is known the heat flux $\delta Q_T = \beta(\delta T_M - \delta T_B)$ on the ocean surface. Then, solving (18), (14), (15) during the interval of time $0 \le t \le t_m$ with use of initial climatic data at t = 0 (the solution of the problem from point 3) the ocean will be adapted step by step to real perturbations getting from the ocean surface.

6. The functional

$$\delta\left(\overline{T}^{\sigma_i}_{M_{t_m}}\right) = -r_T\left(\iint_{\Omega} T_0^* \delta T_0 \, d\Omega + \int_0^{t_m}\int_0^L \beta T^*_{z=0} \delta T_B^{\sigma_i} \, dxdt\right) \qquad \text{(i = 1, 2)}$$

and the perturbed state of the temperature anomaly on the ocean free surface at the time moment $t = t_m$

$$T'^{\sigma_i}_{M_{t_m}} = T^{\sigma_i}_{M_{t_m}} + \delta\left(\overline{T}^{\sigma_i}_{M_{t_m}}\right) \qquad \text{(i = 1, 2).}$$

is calculated.

Here $\delta T_0 = T_0' - T_0$, T_0 is the climatic value, T_0^* is the solution of the problem from the point 5 at moment t_m and T_0^* is the solution of the problem (8), (5), (6) at conditions $T^*_{t_m} = 0$;

7. On this stage the perturbed problem (18), (14), (15) is solving again, in which in the boundary conditions (14) at definition of the heat flux on a free surface, function $T'^{\sigma_i}_{M_{t_m}}$ determined in the item 6 is used, as the initial data at t = 0 solution from point 5 is used, i. e.. $T_0' = T'_{t_m}$ .This procedure can be continued until then, the necessary approximation between calculated and existing fields of anomaly of temperature on a free surface of ocean will not be achieved. Here the received solution T'_{t_m} for the moment of time t_m is used as the initial data for prognostic models in the interval of time $t_m \le t \le T$.

3. Three-dimensional prognostic problem

Now we shall consider 3D problem of the ocean dynamics for which we use results received in the first part of the present work which concerns specification of the field of the temperature anomaly on the ocean free surface. As we have already noted, the quality of the temperature anomaly received as a result of solution of the prognostic baroclinic problem of the ocean dynamics considerably depends on quality of such field.

So, in the closed basin Ω, having depth H and lateral surface Σ, we shall consider the following system of the differential equations, describing dynamics of the baroclinic ocean, written in terms of deviation from standard values of geophysical fields.

$$\begin{aligned}
&u_t + \Lambda_1 u - lv + p_x / \rho_0 = \mu' \Delta u + \nu' u_{zz}, \\
&v_t + \Lambda_1 v + lu + p_y / \rho_0 = \mu' \Delta v + \nu' v_{zz}, \\
&p_z = g\rho, \\
&u_x + v_y + w_z = 0, \\
&T_t + \Lambda_1 T + \gamma_T w = \mu'_T \Delta T + \nu'_T T_{zz}, \\
&S_t + \Lambda_1 S + \gamma_S w = \mu'_S \Delta S + \nu'_S S_{zz}, \\
&\rho = \alpha_T T + \alpha_S S.
\end{aligned} \qquad (19)$$

As boundary and initial conditions for system (19) we shall accept the following

$$\begin{aligned}
&\nu' u_z = -\tau_{xz} / \rho_0 \;,\; \nu' v_z = -\tau_{yz} / \rho_0 \;,\; w = 0, \\
&\nu'_T T_z = Q_T \;,\; \nu'_S S_z = Q_S && \text{on } z = 0; \\
&\text{u=0} \;,\; \text{v=0} \;, \partial T / \partial n = 0 \;,\; \partial S / \partial n = 0 && \text{on } \Sigma; \qquad (20)\\
&u_z = 0, v_z = 0 \;, \text{w=0}, T_z = 0, S_z = 0 && \text{on } z = H; \\
&u = u_0 \;,\; v = v_0 \;,\; T = T_0 \;,\; S = S_0 && \text{at } t = 0. \qquad (21)
\end{aligned}$$

Here u, v, and w are the components of velocity vector $\vec{u}$; p, ρ, T, and S are the deviations of the pressure, density, temperature and salinity of sea water from standard values $\overline{p}(z), \overline{\rho}(z), \overline{T}(z)$ and $\overline{S}(z)$, respectively; $Q_S = S_M(R_n^* + S_n^* - E^*)$, S_M is the deviation of climatic salinity on the see surface, R_n^* is the atmospheric precipitation, S_n^* is snowfall, E^* is the sublimation or evaporation. Thus, it is assumed that, change of salinity is defined by the difference between precipitation and evaporation; τ_{xz}, τ_{yz} are the wind stress components along x and y axis; n is the vector of outer normal to the lateral boundary Σ; $\gamma_T = \overline{T}_z, \gamma_S = \overline{S}_z, \alpha_T = \partial f(\overline{T}, \overline{S}) / \partial \overline{T}, \alpha_S = \partial f(\overline{T}, \overline{S}) / \partial \overline{S}$, where f is the known function of temperature and salinity, and $\rho = f(T, S)$ is the equation of state of marine water, Δ is two-dimensional Laplace operator, $\mu', \mu'_T, \mu'_S, \nu', \nu'_T, \nu'_S$ are the factors of horizontal and vertical viscosity and diffusion, in addition,

$$\mu' = \mu + \delta\mu \,,\, \mu'_{T,S} = \mu_{T,S} + \delta\mu_{T,S} \,,\, \nu' = \nu + \delta\nu \,,\, \nu'_{T,S} = \nu_{T,S} + \delta\nu_{T,S} .$$

It is assumed that the coefficients $\delta\mu$, $\delta\mu_{T,S}$, $\delta\nu$ и $\delta\nu_{T,S}$ are defined on the base of conjugate equations, the theory of small perturbations and the principle of duality of functionals [14, 21, 22]; u_0, v_0, T_0, S_0 are known climatic functions of coordinates. Now, let us assume that we found the solution of the problem (19-21) [1-22] and the received functions u, v and w we will consider as coefficients in operator Λ_1. Then we have

$$\begin{aligned}
&\Lambda' \Phi = \Lambda_1 \Phi + \Lambda'_2 \Phi \quad , \quad \Lambda'_{T,S} \Phi = \Lambda_1 \Phi + \Lambda'_{2,T,S} \Phi , \\
&\Lambda_1 = div \vec{u} \Phi \;,\; \Lambda'_2 \Phi = -\mu' \Delta \Phi - \nu' \Phi_{zz} \;,\; \Lambda'_{2,T,S} \Phi = -\mu'_{T,S} \Delta \Phi - \nu'_{T,S} \Phi_{zz} ,
\end{aligned}$$

where Φ is any function of u, v, T and S. Other designations in the problem (19) - (21) are well-known. In the problem (19) - (21) index (‘), meaning deviations of pressure, density, temperatures and salinity of sea water, are omitted.

Let's assume, that the solution and input data of the problem (19)-(21) have the sufficient smoothness providing existence and uniqueness of the solution of the problem [23-28].

Let's consider vectors φ, F and matrixes A and B

$$\varphi=\begin{bmatrix} u \\ v \\ w \\ p \\ T \\ S \end{bmatrix},\quad F=\begin{bmatrix} u_0 \\ v_0 \\ 0 \\ 0 \\ T_0 \\ S_0 \end{bmatrix},\quad A'=\begin{Vmatrix} \rho_0\Lambda' & -\rho_0 l & 0 & \partial/\partial x & 0 & 0 \\ \rho_0 l & \rho_0\Lambda' & 0 & \partial/\partial y & 0 & 0 \\ 0 & 0 & 0 & \partial/\partial z & -g\alpha_T & -g\alpha_S \\ \partial/\partial x & \partial/\partial y & \partial/\partial z & 0 & 0 & 0 \\ 0 & 0 & g\alpha_T & 0 & g\alpha_T\Lambda'_T/\gamma_T & 0 \\ 0 & 0 & g\alpha_S & 0 & 0 & g\alpha_S\Lambda'_S/\gamma_S \end{Vmatrix},$$

$$B=\begin{Vmatrix} \rho_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g\alpha_t/\gamma_T & 0 \\ 0 & 0 & 0 & 0 & 0 & g\alpha_S/\gamma_S \end{Vmatrix},$$

Then the system of the equations (19) we shall write in the operational form

$$B\frac{\partial\varphi}{\partial t}+A'\varphi=0 \tag{22}$$

and as initial conditions we will assume

$$B\varphi=BF \qquad \text{at} \qquad t=0,$$

and the components of an vector-function φ satisfy the boundary conditions (20).

Let's find now the conjugated operator A'^* in relation to A'. With this purpose we shall consider Lagrangian’ identity

$$\left(\varphi^*,A'\varphi\right)=\left(A'^*\varphi^*,\varphi\right),$$

where $\varphi^*=\left(u^*,\ v^*,\ w^*,\ p^*,\ T^*,\ S^*\right)'$ and scalar product is determined by a ratio

$$\left(g,h\right)=\sum_{i=1}^{3}\iiint_{\Omega} g_i h_i d\Omega .$$

Here g_i and h_i are the components of the vector-function g и h.

Now we shall scalary multiply the equations of the system (19) by $\rho_0 u^*, \rho_0 v^*, w^*, p^*, g\alpha_T T^*/\gamma_T, g\alpha_S S^*/\gamma_S$, accordingly. Then, the received expressions we shall combine and integrate on time from 0 up to the some t_m. Then, we receive

$$\int_0^{t_m}\iiint_\Omega\Big[\rho_0 u^*\left(u_t+\Lambda' u-lv+p_x/\rho_0\right)+\rho_0 v^*\left(v_t+\Lambda' v+lu+p_y/\rho_0\right)+w^*\left(P_z-g\left(\alpha_T T+\alpha_S S\right)\right)+$$

$$+P^* div\vec{u}+\frac{g\alpha_T}{\gamma_T}T^*\left(T_t+\Lambda'_T T+\gamma_T w\right)+\frac{g\alpha_S}{\gamma_S}S^*\left(S_t+\Lambda'_S S+\gamma_S w\right)\Big]d\Omega dt=0\,. \qquad (23)$$

Now we assume, that the functions u^*, v^*, w^*, T^* и S^* satisfy the following conditions

$$\begin{array}{lll} \nu' u_z^*=0,\ \nu' v_z^*=0,\ w^*=0,\ \nu'_T T_z^*=Q_T^*,\ \nu'_S S_z^*=Q_S^* & \text{on } z=0; & \\ u_z^*=0, v_z^*=0, w^*=0, T_z^*=0, S_z^*=0 & \text{on } z=H; & (24) \\ u^*=0,\ v^*=0, \partial T^*/\partial n=0,\ \partial S^*/\partial n=0 & \text{on } \Sigma; & \\ u^*=u_{t_m}^*, v^*=v_{t_m}^*,\ T^*=T_{t_m}^*,\ S^*=S_{t_m}^* & \text{at } t=t_m, & (25) \end{array}$$

where $Q_T^*=\beta T_M^*+f_T^*, Q_S^*=S_M^*(R_n^*+S_n^*-E^*)$, a $f_T^*, u_{t_m}^*, v_{t_m}^*, T_{t_m}^*$ и $S_{t_m}^*$ – are any functions. The left part of the equality (23) we shall transform so that outside of brackets under integral there were functions u, v, w, P, T, and S, and in brackets – the differential ratio containing functions u^*, v^*, w^*, P^*, T^* and S^* [1, 2]. For this purpose with the help of partial integration, formulas Ostrogradsky-Gauss, boundary and initial conditions (20), (21), (24), (25), in view of the continuity equations

$$div\vec{u}=0\,,\ \ div\vec{u}^*=0$$

and some transformations, separate expressions in (23) we will transform to a kind

$$\begin{aligned} &\int_0^{t_m}\left\{\iiint_\Omega\left[\rho_0\left(u^*u_t+v^*v_t\right)+\frac{g\alpha_T}{\gamma_T}T^*T_t+\frac{g\alpha_S}{\gamma_S}S^*S_t\right]d\Omega\right\}dt= \\ &=-\int_0^{t_m}\left\{\iiint_\Omega\left[\rho_0\left(uu_t^*+vv_t^*\right)+\frac{g\alpha_T}{\gamma_T}TT_t^*+\frac{g\alpha_S}{\gamma_S}SS_t^*\right]d\Omega\right\}dt+ \\ &+\iiint_\Omega\left[\rho_0\left(u_{t_m}^*u_{t_m}+v_{t_m}^*v_{t_m}\right)+\frac{g\alpha_T}{\gamma_T}T_{t_m}^*T_{t_m}+\frac{g\alpha_S}{\gamma_S}S_{t_m}^*S_{t_m}\right]d\Omega- \\ &-\iiint_\Omega\left[\rho_0\left(u_0^*u_0+v_0^*v_0\right)+\frac{g\alpha_T}{\gamma_T}T_0^*T_0+\frac{g\alpha_S}{\gamma_S}S_0^*S_0\right]d\Omega, \end{aligned} \qquad (26)$$

$$\begin{aligned} &\int_0^{t_m}\left\{\iiint_\Omega\left[\rho_0\left(u^*div\vec{u}u+v^*div\vec{u}v\right)+\frac{g\alpha_T}{\gamma_T}T^*div\vec{u}T+\frac{g\alpha_S}{\gamma_S}S^*div\overrightarrow{uS}\right]d\Omega\right\}dt= \\ &=-\int_0^{t_m}\left\{\iiint_\Omega\left[\rho_0\left(udiv\vec{u}u^*+vdiv\vec{u}v^*\right)+\frac{g\alpha_T}{\gamma_T}Tdiv\vec{u}T^*+\frac{g\alpha_S}{\gamma_S}Sdiv\vec{u}S^*\right]d\Omega\right\}dt, \end{aligned} \qquad (27)$$

$$\begin{aligned} &\int_0^{t_m}\iiint_\Omega\left[\rho_0\mu'\left(u^*\Delta u+v^*\Delta v\right)+\frac{g\alpha_T\mu'_T}{\gamma_T}T^*\Delta T+\frac{g\alpha_S\mu'_S}{\gamma_S}S^*\Delta S\right]d\Omega dt= \\ &=\int_0^{t_m}\iiint_\Omega\left[\rho_0\mu'\left(u\Delta u^*+v\Delta v^*\right)+\frac{g\alpha_T\mu'_T}{\gamma_T}T\Delta T^*+\frac{g\alpha_S\mu'_S}{\gamma_S}S\Delta S^*\right]d\Omega dt, \end{aligned} \qquad (28)$$

$$\int_0^{t_m}\iiint_{\Omega}\left(\vec{u}^{\,*} gradP + P^* div\vec{u}\right)d\Omega dt = -\int_0^{t_m}\iiint_{\Omega}(\vec{u} gradP^* + p div\vec{u}^{\,*})d\Omega dt, \tag{29}$$

$$\begin{aligned}
&\int_0^{t_m}\iiint_{\Omega}\left[\rho_0 \nu'\left(u^* u_{zz} + v^* v_{zz}\right) + \frac{g\alpha_T \nu_T'}{\gamma_T} T^* T_{zz} + \frac{g\alpha_S \nu_S'}{\gamma_S} S^* S_{zz}\right] d\Omega dt = \\
&= \int_0^{t_m}\iiint_{\Omega}\left[\rho_0 \nu'\left(u u_{zz}^* + v v_{zz}^*\right) + \frac{g\alpha_T \nu_T'}{\gamma_T} T T_{zz}^* + \frac{g\alpha_S \nu_S'}{\gamma_S} S S_{zz}^*\right] d\Omega dt + \\
&+ \int_0^{t_m}\iint_{\Sigma_0}\left[u^* \tau_{xz} + v^* \tau_{yz} + \frac{g\alpha_T}{\gamma_T} T^*\left(\beta T_B + R\right) + \frac{g\alpha_T}{\gamma_T} T f_T^*\right] d\Sigma_0 dt,
\end{aligned} \tag{30}$$

where Σ_0 is the section of a cylindrical surface at a level z = 0.
Let's substitute (26) - (30) in (23), then after corresponding transformations we receive

$$\begin{aligned}
&\int_0^{t_m}\left\{\iiint_{\Omega}\left[\rho_0 u\left(-u_t^* + \Lambda^{\prime *} u^* + l v^* - P_x^*/\rho_0\right) + \rho_0 v\left(-v_t^* + \Lambda^{\prime *} v^* - l u^* - P_y^*/\rho_0\right) + \right.\right. \\
&+ w\left(-P_z^* + g\left(\alpha_T T^* + \alpha_S S^*\right)\right) - P\left(u_x^* + v_y^* + w_z^*\right) + \frac{g\alpha_T}{\gamma_T} T\left(-T_t^* + \Lambda_T'^{*} T^* - \gamma_T w^*\right) + \\
&\left.\left. + \frac{g\alpha_S}{\gamma_S} S\left(-S_t^* + \Lambda_S'^{*} S^* - \gamma_S w^*\right)\right] d\Omega\right\} dt = \\
&= -\iiint_{\Omega}\left[\rho_0\left(u_{t_m}^* u_{t_m} + v_{t_m}^* v_{t_m}\right) + \frac{g\alpha_T}{\gamma_T} T_{t_m}^* T_{t_m} + \frac{g\alpha_S}{\gamma_S} S_{t_m}^* S_{t_m}\right] d\Omega + \\
&+ \iiint_{\Omega}\left[\rho_0\left(u_0^* u_0 + v_0^* v_0\right) + \frac{g\alpha_T}{\gamma_T} T_0^* T_0 + \frac{g\alpha_S}{\gamma_S} S_0^* S_0\right] d\Omega + \\
&+ \int_0^{t_m}\iint_{\Sigma_0}\left[u^* \tau_{xz} + v^* \tau_{yz} + \frac{g\alpha_T}{\gamma_T} T^*\left(\beta T_B + R\right) + \frac{g\alpha_T}{\gamma_T} T f_T^*\right] d\Sigma_0 dt.
\end{aligned} \tag{31}$$

Let us assume that the functions u^*, v^*, w^*, P^*, T^* and S^* satisfy the system of the conjugate equations

$$\begin{aligned}
&-u_t^* + \Lambda^{\prime *} u^* + l v^* - P_x^*/\rho_0 = 0, \\
&-v_t^* + \Lambda^{\prime *} v^* - l u^* - P_y^*/\rho_0 = 0, \\
&-P_z^* + g(\alpha_T T^* + \alpha_S S^*) = 0, \\
&-u_x^* - v_y^* - w_z^* = 0, \\
&-T_t^* + \Lambda_T'^{*} T^* - \gamma_T w^* = 0, \\
&-S_t^* + \Lambda_S'^{*} S^* - \gamma_S w^* = 0
\end{aligned} \tag{32}$$

with boundary and initial conditions (24, (25). Just as in case of the main problem here again we assume performance of conditions of smoothness of solutions of the conjugate problem.

Now, let us multiply the equations of the system (19) by $\rho_0 u^*$, $\rho_0 v^*$, w^*, P^*, $g\alpha_T T^*/\gamma_T$ and $g\alpha_S S^*/\gamma_S$, respectively, then, them we shell combine. After that we multiply the equations of the conjugate system (32), by $\rho_0 u$, $\rho_0 v$, w, P, $g\alpha_T T/\gamma_T$, $g\alpha_S S/\gamma_S$, respectively, and them combine. Then, the results let's subtract one of another and final expression we shall integrate on time from 0 up to t_m and on area Ω. Then, in view of boundary conditions (20) and (24), analogically to (31), after transformations, we shall receive

$$\begin{aligned}
&\iiint_\Omega \left[\rho_0 \left(u^*_{t_m} u_{t_m} + v^*_{t_m} v_{t_m} \right) + \frac{g\alpha_T}{\gamma_T} T^*_{t_m} T_{t_m} + \frac{g\alpha_S}{\gamma_S} S^*_{t_m} S_{t_m} \right] d\Omega - \\
&- \iiint_\Omega \left[\rho_0 \left(u^*_0 u_0 + v^*_0 v_0 \right) + \frac{g\alpha_T}{\gamma_T} T^*_0 T_0 + \frac{g\alpha_S}{\gamma_S} S^*_0 S_0 \right] d\Omega = \\
&= \int_0^{t_m} \iint_{\Sigma_0} \left[u^* \tau_{xz} + v^* \tau_{yz} + \frac{g\alpha_T}{\gamma_T} T^* \left(\beta T_B + R \right) + \frac{g\alpha_T}{\gamma_T} T f^*_T \right] d\Sigma_0 dt.
\end{aligned} \tag{33}$$

Let us assume that

$$f^*_T = \alpha\delta(t - t_m, x - x_0, y - y_0) = \begin{cases} \alpha, & если \quad t = t_m, x = x_0, y = y_0 \\ 0, & если \quad t \neq t_m, x \neq x_0, y \neq y_0, \end{cases},$$

Where

$$\alpha = \frac{v'^3_T \gamma_T}{g\beta}.$$

Then, because of $T_{z=0} \equiv T_M$, we have

$$\int_0^{t_m} \iint_{\Sigma_0} T_M f^*_T d\Sigma_0 dt = \alpha T_{M_{t_m}}(x_0, y_0)$$

and if we assume that

$$u^*_{t_m} = 0, \quad v^*_{t_m} = 0, \quad T^*_{t_m} = 0, \quad S^*_{t_m} = 0, \tag{34}$$

from (33) for the moment $t = t_m$ we have

$$\begin{aligned}
T_{M_{t_m}}(x_0, y_0) = &-r_T \iiint_\Omega \left[\rho_0 \left(u^*_0 u_0 + v^*_0 v_0 \right) + \frac{g\alpha_T}{\gamma_T} T^*_0 T_0 + \frac{g\alpha_S}{\gamma_S} S^*_0 S_0 \right] d\Omega - \\
&- r_T \int_0^{t_m} \iint_{\Sigma_0} \left[u^* \tau_{xz} + v^* \tau_{yz} + \frac{g\alpha_T}{\gamma_T} T_{z=0}{}^* \left(\beta T_B + R \right) \right] d\Sigma_0 \, dt_m,
\end{aligned} \tag{35}$$

where $r_T = \gamma_T / g\alpha_T \alpha$.

In the formula (35) u_0, v_0, T_0, S_0 are given at the initial moment of time, and u^*_0, v^*_0, T^*_0 и S^*_0 are solutions of the conjugate equations (32) at the boundary conditions (24)–(25).

With the help of the functional (35) it is possible to define the values of temperature anomalies on the free surface of the ocean. Moreover, to use them is not practically possible, because for each fixed point $(x_0, y_0) \in \Sigma_0$ it is necessary to solve the conjugate problem (32), (24), (25). With the purpose of simplification of the problem, It is expedient to divide the ocean surface Σ_0 by several parts, i.e. we shall assume $\Sigma_0 = \bigcup_{i=1}^{n} \Sigma_{0i}$, as we shall define average values of temperature anomaly in everyone Σ_{0i}.
For this purpose in (33) we shall assume, that [1]

$$f_T^* = \begin{cases} \dfrac{\alpha}{\Sigma_{0i}} \delta(t - t_m), & if \qquad x, y \in \Sigma_{0i} \\ 0, & out\ of\ the\ domain \qquad . \end{cases} \tag{36}$$

As our problem is specification of values of a field of anomaly of the temperature, given on the ocean free surface in each subarea Σ_{0i}, therefore first of all follows to define average value of anomaly of temperature of century Σ_{0i}. In this connection we shall enter into consideration a designation for average Σ_{0i} on temperature anomaly at the moment of time $t = t_m$ as follows:

$$\overline{T}_{M_{t_m}}^{\Sigma_{0i}} = \frac{1}{\Sigma_{0i}} \iint_{\Sigma_{0i}} T_{M_{t_m}} \, d\Sigma_{0i} \, ,$$

Then, with taking into account (36), we have

$$\int_0^{t_m} \iint_{\Sigma_{0i}} T_M f_T^* d\Sigma_0 dt = \int_0^{t_m} \iint_{\Sigma_{0i}} T_M \frac{\alpha}{\Sigma_0} \delta(t - t_m) d\Sigma_0 dt = \frac{\alpha}{\Sigma_0} \iint_{\Sigma_{0i}} T_{M_{t_m}} d\Sigma_0 = \alpha \overline{T}_{M_{t_m}}^{\Sigma_{0i}} \, ,$$

in view of which and conditions (34) from (33) we shall receive

$$\begin{aligned} \overline{T}_{M_{t_m}}^{\Sigma_{0i}} &= -r_T \iiint_{\Omega} \left[\rho_0 \left(u_0^* u_0 + v_0^* v_0 \right) + \frac{g\alpha_T}{\gamma_T} T_0^* T_0 + \frac{g\alpha_S}{\gamma_S} S_0^* S_0 \right] d\Omega - \\ &- r_T \int_0^{t_m} \iint_{\Sigma_0} \left[u^* \tau_{xz} + v^* \tau_{yz} + \frac{g\alpha_T}{\gamma_T} T^* \left(\beta T_B + R \right) \right] d\Sigma_0 \, dt. \end{aligned} \tag{37}$$

Thus, the problem about specification of values of average temperature anomaly was reduced to the solution of the conjugated problem (32), (24), (25) under condition of (34).
Now, let's consider the perturbed equation system:

$$\begin{aligned} & u_t' + \Lambda_1 u' - l v' + P_x' / \rho_0 - \mu' \Delta u' - \nu' u_{zz}' = 0 \, , \\ & v_t' + \Lambda_1 v' + l u' + P_y' / \rho_0 - \mu' \Delta v' - \nu' v_{zz}' = 0 \, , \\ & P_z' - g\left(\alpha_T T' + \alpha_S S' \right) = 0 \, , \\ & u_x' + v_y' + w_z' = 0 \, , \\ & T_t' + \Lambda_1 T' + \gamma_T w' - \mu_T' \Delta T' - \nu_T' T_{zz}' = 0 \, , \\ & S_t' + \Lambda_1 S' + \gamma_S w' - \mu_S' \Delta S' - \nu_S' S_{zz}' = 0 \end{aligned} \tag{38}$$

at following boundary conditions

$$
\begin{aligned}
&\nu' u'_z = -\tau'_{xz} / \rho_0 \,, \nu' v'_z = -\tau'_{yz} / \rho_0 \,, w' = 0 \,, \\
&\nu' T'_z = \beta\left(T'_M - T'_B\right) - R \,, \nu' S'_z = S'_M \left(R^*_n + S^*_n - E^*\right) && \text{on } z = 0; \\
&u' = 0 \,, v' = 0 \,, \partial T' / \partial n = 0 \,, \partial S' / \partial n = 0 && \text{on } \Sigma; \qquad (39) \\
&u'_z = 0 \,, v'_z = 0 \,, w' = 0 \,, T'_z = 0 \,, S'_z = 0 && \text{on } z = H .
\end{aligned}
$$

As initial we shall accept climatic values

$$
u' = u'_0 \,, v' = v'_0 \,, T' = T'_0 \,, S' = S'_0 \qquad \text{at} \qquad t = 0. \qquad (40)
$$

here $\tau'_{xz} = \tau_{xz} + \delta\tau_{xz} \,, \tau'_{yz} = \tau_{yz} + \delta\tau_{yz} \,, T'_B = T_B + \delta T_B \,, u'_0 = u_0 + \delta u_0 \,, v'_0 = v_0 + \delta v_0 \,, T'_0 = T_0 + \delta T_0 \,,$ $S'_0 = S_0 + \delta S_0 \,.$

Now, let us multiply the equation (38) by the conjugate functions $u^* \rho_0$, $v^* \rho_0$, $w^*, P^*, g\alpha_T T^* / \gamma_T$ and $g\alpha_S S^* / \gamma_S$, respectively, corresponding to the not perturbed (climatic) problem, and let's term by term combine, and the conjugate equations (32) we shall multiply $v' \rho_0 \,, w'$, P', $g\alpha_T T' / \gamma_T$, and $g\alpha_S S' / \gamma_S$ also we shall combine, then results of these operations we shall subtract from each other, the result we shall integrate on time in limits from 0 up to t_m and on the area Ω. Then, analogically to (37) for each subarea $\sum_{0i}$ we shall receive the functional

$$
\begin{aligned}
\overline{T'}^{\Sigma_{0i}}_{M_{t_m}} &= -r_T \iiint_{\Omega} \left[\rho_0 \left(u^*_0 u'_0 + v^*_0 v'_0 \right) + \frac{g\alpha_T}{\gamma_T} T^*_0 T'_0 + \frac{g\alpha_S}{\gamma_S} S^*_0 S'_0 \right] d\Omega - \\
&- r_T \int_0^{t_m} \iint_{\Sigma_0} \left[\tau'_{xz} u^* + \tau'_{yz} v^* + \frac{g\alpha_T}{\gamma_T} T^* \left(\beta T'_B + R \right) \right] d \textstyle\sum_0 dt.
\end{aligned} \qquad (41)
$$

Taking into account, that

$$
\overline{T'}^{\Sigma_{0i}}_{M_{t_m}} = \overline{T}^{\Sigma_{0i}}_{M_{t_m}} + \delta\left(\overline{T}^{\Sigma_{0i}}_{M_{t_m}} \right)
$$

and subtracting from (41) equality (37), we come to functional for average anomaly of temperature for each subarea $\sum_{0i}$ (i = 1,2,...,n) at the moment t_m. Thus, we have

$$
\begin{aligned}
\delta\left(\overline{T}^{\Sigma_{0i}}_{M_{t_m}} \right) &= -r_T \iiint_{\Omega} \left[\rho_0 \left(\delta u_0 u^*_0 + \delta v_0 v^*_0 \right) + \frac{g\alpha_T}{\gamma_T} \delta T_0 T^*_0 + \frac{g\alpha_S}{\gamma_S} \delta S_0 S^*_0 \right] d\Omega - \\
&- r_T \int_0^{t_m} \iint_{\Sigma_0} \left[\delta\tau_{xz} u^* + \delta\tau_{yz} v^* + \frac{g\alpha_T}{\gamma_T} \beta T^* \delta T_B \right] d \textstyle\sum_0 dt
\end{aligned} \qquad (42)
$$

and with using the formula

$$
T'^{\Sigma_{0i}}_{M_{t_m}} = T^{\Sigma_{0i}}_{M_{t_m}} + \delta\left(\overline{T}^{\Sigma_{0i}}_{M_{t_m}} \right) \qquad (43)
$$

we determine the perturbed state of anomaly of temperature in subareas $\sum_{0i}$ (i = 1,...., n) at the moment t_m. We define the perturbed state of anomaly of salinity in the subareas $\sum_{0i}$ (i = 1,...., n) at the moment t_m.

Further the perturbation problem (38)-(40) is salving.

In the boundary conditions on the ocean free surface considered here the specified values for temperature anomaly at the moment t_m, defined under formula (43) are used.

Besides, the specified values of factors of turbulent diffusion and viscosity $\mu' = \mu + \delta\mu$, $\nu' = \nu + \delta\nu$, $\mu'_{T,S} = \mu_{T,S} + \delta\mu_{T,S}$ and $\nu'_{T,S} = \nu_{T,S} + \delta\nu_{T,S}$ are used, that essentially raises adequacy of the results received on the perturbed problem with the existing condition of distribution of geophysical fields beforehand specified moment of time t_m. Thus, certain fields (received as a result of the solution of the perturbation problem) should be used as the initial data at consideration of prognostic model in the range of time from t =t_m till some moment of time t = T.

With taking into account (22), the sequence of realization of the algorithm on preparation of the input information at the initial time $t = t_m$ for the prognostic three-dimensional problem of the ocean dynamics consists from following stages:

1. On the basis of conjugate equations, the theory of small perturbations and principle of duality, we define values $\delta\mu$, $\delta\mu_{T,S}$, $\delta\nu$, $\delta\nu_{T,S}$ and find

$$\mu' = \mu + \delta\mu,\ \mu'_{T,S} = \mu_{T,S} + \delta\mu_{T,S},\ \nu' = \nu + \delta\nu,\ \nu'_{T,S} = \nu_{T,S} + \delta\nu_{T,S};$$

2. The problem

$$\frac{\partial\tilde{\varphi}}{\partial t} + A'\tilde{\varphi} = 0,$$
$$\tilde{\varphi} = 0 \quad \text{at} \quad t=0$$

is solving. Input data used in the boundary conditions are climatic data. The problem is solved to achive a quasistationary state.

3. Within the time interval $0 \le t \le t_m$ the problem

$$\frac{\partial\varphi}{\partial t} + A'\varphi = 0, \qquad \varphi = \tilde{\varphi}_{t_{m_1}} \qquad at \qquad t = 0$$

is solving. The input data used in the boundary conditions on the ocean free surface are climatic data.

4. In the time interval $0 \le t \le t_m$ with consideration subareas $\sum_{0i}$ (i = 1,...,n) n conjugate problems (32), (24), with initial conditions $u^*_{t_m} = 0, v^*_{t_m} = 0, T^*_{t_m} = 0, S^*_{t_m} = 0$ are solved. In the boundary conditions known functions are climatic functions;

5. Let are known the fluxes of heat $\delta Q_T = \beta(\delta T_M - \delta T_B)$ and salt $\delta Q_S = \delta S(R^*_n + S^*_n - E^*)$ on the sea surface and values of $\delta\tau_{xz}$, $\delta\tau_{yz}$, corresponding to wind stress, at any moments of time previous to prognostic time interval. These values within the time interval $0 \le t \le t_m$ are determined by solution of problems of atmosphere and ocean dynamics or as a result of direct measurements.

Then, if we solve the problem (38) – (40) in the range of time $0 \le t \le t_m$ with using of climatic initial data at t = 0 (the solution of the problem from point 3) step by step, the ocean will adapt to the real perturbations on the sea surface;

6. The functional

$$\delta\left(\overline{T}_{M_{t_m}}^{\Sigma_{0i}}\right) = -r_T \iiint_{\Omega} \left[\rho_0 \left(\delta u_0 u_0^* + \delta v_0 v_0^* \right) + \frac{g\alpha_T}{\gamma_T} \delta T_0 T_0^* + \frac{g\alpha_S}{\gamma_S} \delta S_0 S_0^* \right] d\Omega -$$
$$- r_T \int_0^{t_m} \iint_{\Sigma_0} \left[\delta\tau_{xz} u^* + \delta\tau_{yz} v^* + \frac{g\alpha_T}{\gamma_T} \beta T_{z=0}^{\ \ *} \delta T_B \right] d\Sigma_0 dt,$$

and temperature anomaly on the ocean free surface

$$T'^{\Sigma_{0i}}_{M_{t_m}} = T^{\Sigma_{0i}}_{M_{t_m}} + \delta\left(\overline{T}^{\Sigma_{0i}}_{M_{t_m}}\right)$$

are calculating.

Here $\delta u_0 = u_0' - u_0$, $\delta v_0 = v_0' - v_0$, $\delta T_0 = T_0' - T_0$, $\delta S_0 = S_0' - S_0$, u_0, v_0, T_0, S_0 are climatic values, and u_0' , v_0' , T_0' , S_0' are solutions of the problem from point 5 at the moment t_m;

7. The perturbation problem is salving, where in boundary conditions at definition of heat fluxes on the free surface the function $T'^{\Sigma_{0i}}_{M_{t_m}}$, defined in the previous paragraph, is used. As the initial data at t = 0 the solution of the problem from the point 5 is assumed, i.e., $u_0' = u_{t_m}'$, $v_0' = v_{t_m}'$, $T_0' = T_{t_m}'$ and $S_0' = S_{t_m}'$. Further we return to the solution of the problem from point 3. Thus, we cycle calculations for the subsequent specification of values of fields of temperature anomalies on the free surface of the ocean at the moment t_m.

At this stage the received solution of the problem for time moment t_m, i.e., u_{t_m}' , v_{t_m}' , T_{t_m}' and S_{t_m}' are used as the initial data for solution of the prognostic model of dynamics of the baroclinic ocean in the range of time $t_m \le t \le T$.

Analogically there is possible to receive functionals for calculation of average anomaly of the salinity $\overline{S}^{\Sigma_{0i}}_{M_{t_m}}$ and $\overline{S'}^{\Sigma_{0i}}_{M_{t_m}}$. We have

$$\left(\overline{S}^{\Sigma_{0i}}_{M_{t_m}}\right) = -r_S \iiint_{\Omega} \left[\rho_0 \left(u_0 u_0^* + v_0 v_0^* \right) + \frac{g\alpha_T}{\gamma_T} T_0 T_0^* + \frac{g\alpha_S}{\gamma_S} S_0 S_0^* \right] d\Omega -$$
$$- r_s \int_0^{t_m} \iint_{\Sigma_0} \left[\tau_{xz} u^* + \tau_{yz} v^* + \frac{g\alpha_T}{\gamma_T} T^* (\beta T_B + R) + \frac{g\alpha_S}{\gamma_S} S_M S^* \left(R_n^* + S_n^* - E^* \right) \right] d\Sigma_0 dt \quad (44)$$

and

$$\left(\overline{S'}^{\Sigma_{0i}}_{M_{t_m}}\right) = -r_S \iiint_{\Omega} \left[\rho_0 \left(u_0' u_0^* + v_0' v_0^* \right) + \frac{g\alpha_T}{\gamma_T} T_0' T_0^* + \frac{g\alpha_S}{\gamma_S} S_0' S_0^* \right] d\Omega -$$
$$- r_s \int_0^{t_m} \iint_{\Sigma_0} \left[\tau_{xz}' u^* + \tau_{yz}' v^* + \frac{g\alpha_T}{\gamma_T} T^* (\beta T_B' + R) + \frac{g\alpha_S}{\gamma_S} S_M' S^* \left(R_n^* + S_n^* - E^* \right) \right] d\Sigma_0 dt, \quad (45)$$

where $r_S = \gamma_s / g\alpha_s \alpha$, $\alpha = v_s'^3 \gamma_s / g\beta$.

In this case in boundary conditions (24) on the ocean surface we assume that

$$Q_T^* = \beta T_M^* \ \text{ и } \ Q_S^* = f_S^* \ \text{ at } \ z = 0,$$

where f_S^* is defined analogically to f_T^*.

Taking into account (44) and (45), analogically to (42), we obtain

$$\delta\left(\overline{S}_{M_{t_m}}^{\Sigma_{0i}}\right) = -r_S \iiint_{\Omega} \left[\rho_0 \left(\delta u_0 u_0^* + \delta v_0 v_0^*\right) + \frac{g\alpha_T}{\gamma_T} \delta T_0 T_0^* + \frac{g\alpha_S}{\gamma_S} \delta S_0 S_0^* \right] d\Omega -$$

$$- r_S \int_0^{t_m} \iint_{\Sigma_0} \left[\delta\tau_{xz} u^* + \delta\tau_{yz} v^* + \frac{g\alpha_T}{\gamma_T} \delta T_B T^* + \frac{g\alpha_S}{\gamma_S} \delta S_M S^* \left(R_n^* + S_n^* - E^*\right) \right] d\Sigma_0 dt$$

and on a formula

$$S'^{\Sigma_{0i}}_{M_{t_m}} = S^{\Sigma_{0i}}_{M_{t_m}} + \delta\left(\overline{S}^{\Sigma_{0i}}_{M_{t_m}}\right)$$

we define the perturbed state for a salinity anomaly in subareas $\sum_{0i}$ (i = 1,...., n) at the time moment t_m.

3. Conclusion

The algorithm on the specification of temperature anomalies on the ocean sea surface, considered in the present article, may be used in numerical calculations of the ocean dynamics.

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References

[1] Marchuk G. I. Numerical solution of problems of atmospheric and oceanic dynamics. Leningrad, Gidrometeoizdat, 1974, 303 p (in Russian).
[2] Marchuk G. I. Mathematical modeling in the environmental problem. Moscow, Nauka, 312 p (in Russian).
[3] Marchuk G. I., Zalesnyi V. B. Numerical model of large-scale circulation in the World ocean. In: Numerical methods of calculation of oceanic currents. Novosibirsk, 1974, p. 3-20 (in Russian).
[4] Marchuk G. I., Kordzadze A. A., Skiba Yu. N. Calculation of major hydrological fields of the Black Sea on the basis of the splitting method. Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 1975, t. 11, N 4, p. 379-393.
[5] Marchuk G. I., Kuzin V. I. On method of finite elements in modeling of oceanic currents. Novosibirsk, VTs SO AN SSSR, 1974, Preprint N 14, 82 p (in Russian).
[6] Marchuk G. I., Kordzadze A. A., Zalesnyi V. B. Problems of mathematical modeling of sea and ocean currents. In: Differential and Integral Equations: Boundary-Value Problems. Tbilisi, 1979, pp. 99-151 (in Russian).
[7] Marchuk G. I., Kordzadze A. A. Numerical modeling of sea dynamics on the base of splitting method. In: Numerical modeling of climate of the World ocean. OVM AN SSSR, Moscow, 1986, p. 151-163 (in Russian).
[8] Marchuk G. I., etc. Mathematical models of ocean circulation. Novosibirsk, Nauka, 1980, 288 p (in Russian).
[9] Manabe C., Brayan K. Climate and circulation of the ocean. Leningrad, Gidrometeoizdat, 1972, 192 p (in Russian).
[10] Tamsaly R. E. Modeling of dynamics and structure of the Baltic Sea. Riga, Zvaygzne, 1979, 151 p.

[11] Ivanov Yu. A. Large-scale and synoptic variability of fields in the ocean. Moscow, Nauka, 1981, 188 p.
[12] Bubnov M. A. Mathematical aspects of modeling of ghghghg and circulation in the baroclinic ocean. Novosibirsk, VTs SO AN SSSR, 1984, 152 p (in Russian).
[13] Kuzin V. I. Method of finite elements in modeling of oceanic currents. Novosibirsk, VTs SO AN SSSR, 1985, 189 c (in Russian).
[14] Kordzadze A. A. Mathematical modeling of sea currents (theory, algorithms, numerical experiments), OVM AN SSSR, Moscow, 1989, 218 p (in Russian).
[15] Kordzadze, A. A., Demetrashvili, D. I., Surmava, A. A. Numerical modeling of hydrophysical fields of the Black Sea under the conditions of alternation of atmospheric circulation processes, Izvestiya RAS, Atmospheric and Oceanic Physics, 2008, v. 44, N 2, p. 213-224.
[16] Kordzadze A. A., Demetrashvili D. I. About coupled regional modeling system the Black Sea-atmosphere. Journal of Environmental Protection and Ecology (JEPE), 2011, v. 12, p. 317-326.
[17] Kordzadze A. A., Demetrashvili D. I. Operative forecast of hydrophysical fields in the Georgian Black Sea coastal zone within the ECOOP. Ocean Sci. Discuss., 8, 2011, p.397-433, www.ocean.sci-discuss.net/8/397/2011/doi:10.5194/osd-8-397-2011.
[18] Demyshev S. G., Korotaev G. K. Numerical modeling of the seasonal trend of synoptic variability of the Black Sea. Izv. Akad. Nauk, Fiz. Atmos. Okeana, 32, 1996, pp.108-116 (in Russian).
[19] Blumberg A. F., Mellor G. L. Description of a three–dimensional coastal ocean circulation model. In : Three–dimensional Coastal Ocean Models. Washington : AGU, 1987, V.4, 208p.
[20] Korotaev G. K., Eremeev V. N. Introduction in operative oceanography of the Black Sea. Sevastopol, HPC "EKOCI-Gideofizika" 382 p (in Russian).
[21] Marchuk G. I., Kordzadze A. A. Perturbation theory and the formulation of inverse problems of ocean dynamics. Trudy Tbilisskogo Universiteta, Mathematics, Mechanics, Astronomy. Tbilisi, t. 259 (1920), pp. 49-65 (in Russian).
[22] Kordzadze A., Kvaratskhelia D., Demetrashvili D. On the specification of the eddy viscosity coefficient in the Black Sea. J. Georgian Geophys. Soc.,1998, v.3B, pp.59–65.
[23] Kordzadze A. A. About uniqueness of the solution of one problem of ocean dynamics. Dokl. AN SSSR, 1974, t. 219, N 4, pp. 856-859.
[24] Kordzadze A. A. To uniqueness of the solution of quasilinear problems of ocean dynamics – In: Some problems of computing and applied mathematics. Novosibirsk, Nauka, 1975, pp. 77 – 89 (in Russian).
[25] Kordzadze A. A. Resolvability of three-dimensional non-stationary problems of the baroclinic ocean. In: Mathematical Modeling of Ocean Dynamics (Mathematical Modeling of Dynamics of Atmosphere and Ocean), p.1, Novosibirsk, 1980, pp. 114 – 139 (in Russian).
[26] Kordzadze A. A. Mathematical problems of solving problems of ocean dynamics. Novosibirsk, VTs SO AN SSSR, Novosibirsk, 1982, 148 p (in Russian).
[27] Sukhonosov V. I. On the corrections as a whole of a three-dimensional problem of ocean dynamics. In: Mechanics of Inhomogeneous Continuous mediums. Novosibirsk. VTs SO AN SSSR, Novosibirsk, 1981, N 52, pp. 37-53 (in Russian).
[28] Bubnov M. A. About resolvability of quasistationary linear dynamics problems of the baroclinic ocean dynamics. In: Mathematical Modeling of Ocean Dynamics (Mathematical modeling of dynamics of atmosphere and ocean), p. 1, Novosibirsk, 1980, pp. 59 – 75 (in Russian).

О подготовке начальных данных для прогностической задачи динамики бароклинного океана

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Резюме

Как известно, качество гидрофизических полей, полученных в результате реализации прогностической модели динамики бароклинного океана, значительно зависит от качества входных данных. В настоящей работе, на основе сопряжённых уравнений и теории возмущении предлагается алгоритм уточнения данных наблюдений о нестационарных процессах, участвующих в граничных условиях на свободной поверхности моря для прогностической задачи динамики океана. С целью удобства, рассмотренный в настоящей работе алгоритм о подготовке начальных данных, сначала рассматривается на примере двумерного уравнения переноса-диффузии субстанции в вертикальной плоскости xoz, а затем для трёхмерной задачи динамики бароклинного океана.

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რეზიუმე

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