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# Thy Consistent Criterion for Homogeneous Gaussion Fields Statistical Structures

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## *ABSTRACT*

In the paper there are discussed Gaussian fields Statistical Structures $\{E, S, \mu_h,\ h \in H\}$ in Banach Space of measures, we prove necessary and sufficient conditions for existence of such criterion in Banach space of measures.

***Key words***: consistent criterion, orthogonal, straggly separable Statistical structures.
Classification cocles 62H05, 62H12.

Let there is given (E,S) measurable space and on this space there given $\{\mu_h,\ h \in H\}$ family of probabilitg measures defined on S, The H set of hypotheses. Thy following definitions are taken from thy works ([1]-[5]).

Definition 1. A statistical structure is is called object $\{E, S, \mu_h,\ h \in H\}$

Definition 2. A statistical structure $\{E, S, \mu_h,\ h \in H\}$ is called orthogonal (singular) (O) if thy family of probability measures $\{\mu_h,\ h \in H\}$ are pairwise singular measures.

For $\{\mu_h,\ h \in H\}$ be probability measures defined on thy measurable space (E,S). For each $h \in H$ denote by $\overline{\mu}_h$ thy completion of thy measure $\mu_h$ and denote by dom($\overline{\mu}_h$) thy $\sigma$-algebra of all $\overline{\mu}_h$-measurable subsets of E.

Let $S_1 \bigcap\limits_{h \in H} \mathrm{dom}(\overline{\mu}_h)$.

Definition 3. A statistical structure $\{E, S_1, \overline{\mu}_h,\ h \in H\}$ is called strongly separable if there exists thy family of $S_1$ - measures sets $\{Z_h,\ h \in H\}$ such that the relations are fulfilled:

1) $\overline{\mu}_h(Z_a) = 1, \quad \forall h \in H;$
2) $Z_{h_1} \cap Z_{h_2} = \varnothing \quad \forall h \in H;$
3) $\bigcup\limits_{h \in H} Z_h = E.$

Definition 4. We consider the concept of the hypothesis as any assumption that determines the form of the distribution of the population.

Let H be set of hypotheses and B(H) be $\sigma$-algebra of subsets of H which contains all finite subsets of H.

Definition 5. We will say that the statistical structure $\{E, S\ , \mu_h\ ,\ h \in H\}$ admits a consistent criterion (CC) for testing hypothesis if there exists at least one measurable mapping

$\delta:(E,S)\to(H,B(H))$,

Such that

$\mu_h(\{x:\delta(x)=h\})=1, \quad \forall h\in H.$

Remark 1. The notion and corresponding construction of consistent criterion for testing hypotheses was introduced and sudid by Z.Zerakidze (see [5]).

Let $M^\sigma$ be real linear space of all alternating finite measures on S.

Definition 6. A linear subset $M_B \subset M^\sigma$ is called a Banach space of measures if:

1) A norm can be defined on $M_B$ so that $M_B$ will be a Banach space with respect to this norm and for are orthogonal measures $\mu,\nu\in M_B$ and real number $\lambda\neq 0$ the enequality $\|\mu+\lambda\nu\|\geq\|\mu\|$ is fulfielled;
2) If $\mu\in M_B$, $|f(x)|\leq 1$ then $\nu_f(A)=\int_A f(x)\mu(dx)\in M_B$ and $\|\nu_f\|\geq\|\mu\|$ ;
3) If $\nu_n\in M_B$, $\nu_n\geq 0, \nu_n(E)<+\infty$, $n=1,2,...$ and $\nu_n\downarrow 0$ then for any linear functional $l^*\in M_B^*$ $\lim\limits_{n\to\infty} l^*(\nu_n)=0$, where $M_B^*$ conjugate to $M_B$ linear space.

Remark 2. The definition and construction of the Banach Space of measures is studied Z.Zerakidze (see [4]).

Defination 7. Let H some set if indexes and $M_{B_h}$ Banach space $\forall h\in H$. We set

$$\left\{\{x_h\}_{h\in H}, x_h\in M_{B_h}, \sum_{h\in H}\|x_h\|_{M_{B_h}}<\infty\right\}.$$

Then the $M_B$ with norm $\|\{x_h\}_{h\in H}\|=\sum_{h\in H}\|x_h\|_{M_{B_h}}<\infty$ is the Banach spase. It is tceeled the direct sum of Banach spaces $M_{B_h}$ and denoted so $M_B=\oplus_{h\in H} M_{B_h}$.

The following theorem has also been proved in the paper (see [4]).

Theorem 1. Let $M_B$ be a Banach space of measures, then in $M_B$ there exists a family of pairwise orthogonal probability measures $\{\mu_h, h\in H\}$ such that $M_B=\oplus_{h\in H} M_{B_h}$, where $M_{B_h}$ is Banach space of elements $\nu$ of the norm:

$$\nu(B)=\int_B f(x)\mu_h(dx), \quad B\in S, \quad \int_E|f(x)|\mu_h(dx)<\infty, \quad \|\nu\|_{M_{B_h}}=\int_E|f(x)|\mu_h(dx)$$

Let $t=(t_1,t_2,.....,t_n)\in T$, where T be closed boundet subset of $R^n$, $\xi_h(t,\omega)$, $t\in T$, $\forall h\in H$ Gaussian real homogenous field on T with rero means $E[\xi_h(t,\omega)]=0$, $\forall h\in H$, and correlation function $E[\xi_h(t,\omega),\xi_h(s,\omega)]=R_h(t-S)$, $t,S\in T, h\in H.$.

Let $\{\mu_h, \ h\in H\}$ be the corresponding probability measures given on S and $f_h(\lambda), \lambda\in R^n$, $\forall h\in H$ be spectral densities.

We be called the Fourier transformation generation Fouer transformation. Let

$\iint_{R^n}\frac{|b_{h,h'}(\lambda,\mu)|^2}{f_h(\lambda)f_h(\mu)}d\lambda d\mu=+\infty, \quad \forall h,h'\in H,$ where $b_{h,h'}(\lambda,\mu)$, $\lambda,\mu\in R^n$, $\forall h,h'\in H$ the generalization Fouer transformation of the following function

$b_{h,h'}(s,t)=R_h(s,t)-R_{h'}(s,t), \ s,t\in T, \ \forall h,h'\in H.$

Then the corresponding probability measures $\mu_h$ and $\mu_{h'}$ are pairwise orthogonal $\forall h, h' \in H$ (see [6]) and $\{E, S, \mu_h,\ h \in H\}$ are Gaussian orthogonal homogeneous fields statistical structures. Next, we consides S – measurable $g_h(x),\ \forall h \in H$, functions such that

$$\sum_{h \in H} \int_E |g_h(x)| \mu_h(dx) < +\infty$$

Let $M_B$ the set measures defined by formula $\nu(B) = \sum_{h \in I_h} \int_B |g_h(x)| \mu_h(dx)$, where $I_h \subset H$ a countable subsets in H and $\sum_{h \in I_h} \int_E |g_h(x)| \mu_h(dx) < \infty$, define a norm on $M_B$ by formula $\|\nu\| = \sum_{h \in I_h} \int_E |g_h(x)| \mu_h(dx)$, then $M_B$ is a Banach space of Measures and $M_B = \oplus_{h \in H} M_{B_h}$, where $M_{B_h}$ is Banach space of elements the norm $\nu(B) = \int_B g(x) \mu_h(dx),\ \ B \in S,\ \int_E |g_h(x)| \mu_h(dx) < \infty,$ with the norm on $M_{B_h}$, $\|\nu\|_{M_{B_h}} = \int_E |g_h(x)| \overline{\mu}_h(dx)$

Let E is the complete separable metric space and $S_1 = \bigcap_{h \in H} dom(\overline{\mu}_h)$ the Borel $\sigma$ - algebra in E and card $H \le C$.

Then the following theorem holds:

Theorem 2. In order that the orthogonal Homogeneous Gaussian Fields statistical structure $\{E, S_1, \overline{\mu}_h,\ h \in H\}$, card $H \le C$ admits a consistent criterion for testing hypothesis in the theory (ZFC)&(MA) it is necessary and sufficient that the correspondence $f \to l_f$ defined by the equality $\int_E |g_h(x)| \overline{\mu}_h(dx) = l_f(\mu_h)$ is one-to-one. Here $l_f$ is a linear continuous functional on $M_B$, $f \in F(M_B)$

Denote by F= $F(M_B)$ the set or real functions f for which $\int_E f(x) \overline{\mu}_h(dx)$ is defined $\forall \overline{\mu}_h \in M_{B'}$

.

Prof Necessity. The existence of a constituent criterion for testing hypothesis $\delta : (E, S_1) \to (H, B(H))$ Implies that $(\forall h)(h \in H \to \overline{\mu}_h(\{x : \delta(x) = h\}) = 1$

Setting $x_h = \{x : \delta(x) = h\}$ for $\forall h \in H$, we get:

1) $\overline{\mu}_h(x_h) = \overline{\mu}_h(\{x : \delta(x) = h\}) = 1, \forall h \in H$

2) $x_h \bigcap x_{h'} = \varnothing,\ \ \forall h = h',\ h = h',\ h, h' \in H;$ because $x_h = (\{x : \delta(x) = h\}) \bigcap (\{x : \delta(x) = h'\}) = \varnothing$;

3) $\bigcup_{h \in H} x_h = \{x : \delta(x) \in H\} = E$

Therefore a statistical structure $\{E, S, \overline{\mu}_h,\ h \in H\}$ is strongly separable, so there exist $S_1$ - measurable sets $\{x_h\}, \forall h \in H$ such that

$$\overline{\mu}_h(x_{h'}) = \begin{cases} 1, & \text{if } h = h' \\ 0, & \text{if } h \ne h' \end{cases}$$

We put the linear contiuous functional $l_{x_h}$ into the correspondence to a function $I_{x_h}(x) \in F(M_B)$ by the formula: $\int_E I_{x_h}(x) \overline{\mu}_h(dx) = l_{x_h}(\overline{\mu}_h) = \|\overline{\mu}_h\|_{M_{B_h}}$

We put the linear continuous functional $l_{\tilde{f}_1}$ into the correspondence to a function $\tilde{f}_1(x) = f_1(x) I_{x_h}(x)$.

Then for $\bar{\mu}_{h'} = M_B(\bar{\mu}_h)$

$$\int_E \tilde{f}_1(x)\bar{\mu}_{h'}(dx) = \int_E f_1(x) I_{x_h}(x)\bar{\mu}_{h'}(dx) = \int_E f(x) f_1(x) I_{x_h}(x)\bar{\mu}_h(dx) = l_{\tilde{f}_1}(\bar{\mu}_{h'}) = \|\bar{\mu}_{h'}\|_{M_{B_h}}$$

Let $\sum$ be the collection of extensions of functional satisfying the condition $l_f \le p(x)$ on those supspaces where they are defined.

Let us intraduce on $\sum$ a partial ordering having assumed $l_{f_1} < l_{f_2}$ if $l_{f_2}$ is defined on large set then $l_{f_1}$ and $l_{f_1} = l_{f_2}$ there where both of them are defined.

Let $\{l_{f_h}\}_{h\in H}$ be a linear ordered subset in $\sum$. Let $M_{B_h}$ be the subspace on wich $l_{f_h}$ is defined. Define $l_f$ on $\bigcup_{h\in H} M_{B_h}$ having assumed $l_f(\bar{\mu}) < l_{f_h}(\bar{\mu})$ if $\bar{\mu} \in M_{B_h}$.

It is obvious, that $l_{f_h} < l_f$. Since any linearly ordered subset in $\sum$ has an upper bound by vintue of Chorn's lemma $\sum$ contains a maximal element $\lambda$ defined on some set $X'$ satisfying the condition $\lambda(x) \le p(x)$ for $x \in X'$. But $X'$ must coincide with the entire space $M_B$ because otherwise. We could extended $\lambda$ to a wider space by adding as above one more dimension. This contradicts the maximality of $\lambda$ hence $X' = M_B$. Therefore the extension of the functional is defined everywhere. The extension of the functional is defined everywhere.

It we put the linear continuous functional $l_f$ into correspondence to the function

$f(x) = \sum_E g_h(x) I_{x_h}(x) \in F(M_B)$ then obtain $\int_E f_1(x)\bar{\mu}_h(dx) = \|\bar{\mu}\| = \sum_{h\in H} \|\bar{\mu}_h\|_{M_{B_h}}$, where

$\bar{\mu}(B) = \sum_{h\in H} \int_B g_h(x)\bar{\mu}_h(dx), \quad B \in S$.

From this discussion it follows that the above-indicated correspondence puts some function $f \in F(M_B)$ into correspondence to each $\psi_f \in M_B$ if we identify in $F(M_B)$ the functions coinciding with respect to the measure $\bar{\mu}_h, h \in H$, then this correspondence will be bijective.

The necessity is proved.

Sufficiency. For $f \in F(M_B)$ we define linear continuous functional by the equality $\int f(x)\bar{\mu}(dx) = l_f(\bar{\mu})$.

Denote $I_f$ a countable subset in $H$ for wich $\int_E f(x)\bar{\mu}_h(dx) = 0$ for $h \notin I_f$

Let us consider functional $l_{f_h}$ on $M_{B_h}$ to which corresponds.

Then for $\bar{\mu}_{h_1}, \bar{\mu}_{h_1} \in M_{B_h}$ have

$\int_E f_{h_1}(x)\bar{\mu}_{h_2}(dx) = l_{f_{h_1}}(\bar{\mu}_{h_2}) = \int_E f_1(x) f_2(x)\bar{\mu}_{h_1}(dx) = \int_E f_{h_1}(x)\mu_{h_1}(dx)$ therefore $f_{h_1} = f_1$ with respect measure $\bar{\mu}_{h_1}$. Let $f_h > 0$ a. e. wich Respect to the measure $\bar{\mu}_h$ and $\int_E f_h(x)\bar{\mu}_h(dx) < \infty$,

$\bar{\mu}_h(c) = \int_c f_h(x)\bar{\mu}_h(dx)$, then

$$\int_E f_h(x)\tilde{\mu}_h(dx) = l_{f_h}(\tilde{\mu}_{h'}) = 0 \;\; \forall h \neq h'.$$

Denote $C_h = \{x : f_h(x) > 0\}$, then $\int_E f_h(x)\bar{\mu}_{h'}(dx) = 0 \;\; \forall h \neq h'.$

Heuce it follows that $\bar{\mu}_h(C_{h'}) = 0, \quad \forall h \neq h'.$ On the other hand $\bar{\mu}_h(E - C_h) = 0,$ therefore the statistical structure $\{E, S_1, \overline{\mu_h},\ h \in H\}$ is weakly separable. We represent $\{\overline{\mu_h},\ h \in H\}$, $\mathrm{Card} H \leq C$ as an inductive sequence $\{\bar{\mu}_h h < \omega_1\}$, where $\omega_1$ denotes the first ordinal number of the power of the set H.

Since the statistical structure $\{E, S_1, \overline{\mu_h},\ h \in H\}$ is weakly separable, there exists the family of $S_1$- measurable sets $\{X_h,\ h \in H\}$ such that for all $h \in [0, \omega_1]$ we have:

$$\bar{\mu}_h(x_{h'}) = \begin{cases} 1, \text{ if } h = h' \\ 0, \text{ if } h \neq h' \end{cases}$$

We define $\omega_1$ sequence of parts of the space $Z_h$ such that the following relations hold:

1) $Z_h$ is borel subset of E for all $h < \omega_1$ ;
2) $Z_h \subset X_h$ for all $h < \omega_1$ ;
3) $Z_h \bigcap Z_{h'} = \varnothing$ for all $h < \omega_1$ , h, $h' < \omega_1$ , $h \neq h'$ ;
4) $\bar{\mu}_h(Z_h) = 1$ for all $h <= \omega_1$ .

Suppose that $Z_{h_0} = X_{h_0}$. Suppose further that the partial sequence $\{Z_{h'}\}_{h'<h}$ is already defined for h $h < \omega_1$.

It is clear that $\mu^{\bullet}(\bigcup_{h'<h} Z_{h'}) = 0$ (see [3] ). Thus there exists a Borel subset $Y_h$ of the space E such that the following relations valid: $\bigcup_{h'<h} Z_{h'} \subset Y_h$ and $\mu^{\bullet}(Y_h) = 0$

Assuming that $Z_h = X_h - Y_h$, we construct the $\omega_1$ sequence $\{Z_h\}_{h<\omega_1}$ of disjunctive measurable subsets of the space E. Therefore $\bar{\mu}_h(Z_h) = 1, \forall h < \omega_1$ and the statistical structure $\{E, S_1, \overline{\mu_h},\ h \in H\}$, $\mathrm{Card} H \leq C$ is strongly separable because there exists a family of elements of the $\sigma$ - algebra $S_1 = \bigcap_{h \in H} \mathrm{dom}(\bar{\mu}_h)$ such that:

1) $\bar{\mu}_h(Z_h) = 1, \quad \forall h \in H;$
2) $Z_h \bigcap Z_{h'} = \varnothing \;\; \forall h, h' \; h \neq h' \in H$;
3) $\bigcup_{h \in H} Z_h = E$ . .

For $x \in E$, we put $\delta(x) = h,$ whore h is the unique hypothesis from the set H for which $x \in Z_h$. The existence of such a unique hypothesis H can be proved using conditions 2), 3).

Now let $Y \in B(H)$ . Then $\{x : \delta(x) \in Y\} = \bigcup_{h \in H} Z_h$ .

We must show that $\{x:\delta(x)\in Y\}\in dom(\overline{\mu}_{h_0})$ for each If $h_0 \in Y$, then

On the one hand, from the validity of conditions 1), 2), 3) it follows that

$$Z_{h_0}\in S_1=\bigcap_{h\in H} dom(\overline{\mu}_h)\subseteq dom(\overline{\mu}_{h0})$$

.

On the other hand, the validity of the condition $\bigcup_{h\in Y-\{h_0\}} Z_h \subseteq (E-Z_{h_0})$

Implies that

$$\overline{\mu}_{h0}(\bigcup_{h\in Y-\{h_0\}} Z_h)=0$$

The last equality yields that $\bigcup_{h\in Y} Z_h \in dom(\overline{\mu}_{h0})$.

Since $dom(\overline{\mu}_{h0})$ is $\sigma$ - algebra, we deduce that $\{x:\delta(x)\in Y\}=(Z_{h_0})\bigcup(\bigcup_{h\in Y-\{h_0\}} Z_h)\in dom(\overline{\mu}_{h0})$

.

If $h_0 \notin Y$ , then

$$\{x:\delta(x)\in Y\})=\bigcup_{h\in Y} Z_h \subseteq (E-Z_{h_0})$$

and we conclude that $\overline{\mu}_{h0}(\{x:\delta(x)\in Y\})=0$ .

The last relation implies that $\{x:\delta(x)\in Y\})\in dom(\overline{\mu}_{h0})$

.

Thus we have shown the validaty of the relation $\{x:\delta(x)\in Y\})\in dom(\overline{\mu}_{h0})$ for an arbitrary $h_0 \in H$.

Heuce $\{x:\delta(x)\in Y\}\in \bigcap_{h\in H}(dom(\overline{\mu}_h))=S_1$

.

We have shown that the nap: $\delta:(E,S_1)\to(H,B(H))$

Is measurable map and we ascertain that $\overline{\mu}_h(\{x:\delta(x)=h\})=\overline{\mu}_h(Z_h)=1, \ \forall h\in H.$

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Theorem is proved.

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# თანმიმდევრული კრიტერიუმები ჰომოგენური გაუსის ველების სტატისტიკურ სტრუქტურებისთვის

ზ. ზერაკიძე, დ. ქირია, თ. ქირია, ი. ლოლაძე

### რეზიუმე

ნაშრომში განიხილება გაუსის ველები სტატისტიკური სტრუქტურებისთვის ბანახის ზომათა სივრცეში, ვამტკიცებთ აუცილებელ და საკმარის პირობებს ამ კრიტერიუმების არსებობისათვის ბანახის ზომათა სივრცეში.

# Последовательный критерий для статистических структур однородных Гауссовых полей

З. Зеракидзе, Д. Кирия, Т. Кирия, И. Лоладзе

### Резюме

В статье обсуждаются статистические структуры Гауссовских полей в Банаховом пространстве мер, доказываются необходимые и достаточные условия существования такого критерия в Банаховом пространстве мер.