Journal of the Georgian Geophysical Society, e-ISSN: 2667-9973, p-ISSN: 1512-1127
Physics of Solid Earth, Atmosphere, Ocean and Space Plasma, v. 24(1), 2021, pp. 11-17

The Sharle Statistical Structure

[1]Zurab S. Zerakidze, [2]Jemal K. Kiria, [2]Tengiz V. Kiria

[1]Gori State Teaching University
e-mail: zura.zerakidze@mail.ru
[2]M. Nodia Institute of Geophysics of I. Javakhishvili Tbilisi State University
e-mail: kiria51@yahoo.com

ABSTRACT

In this paper the consistent criteria for testing Hypothesis for the Sharle statistical structure are defined. It is shown that the necessary and sufficient conditions for the existence of these critical are considered. Also the necessary and sufficient conditions for the existence of such criteria for the Sharle strongly statistical structure.

Key words: Consistent criterion, statistical structures.

I. The Sharle statistical structure.

Definition 1.1 we will say that X random value is (see [1]-[6]) the Sharle distribution if this density given by formula

$$f_{sh}(x) = f(x) + \frac{1}{\sigma}\left[\frac{S_k(X)}{6} \cdot Z_u \cdot (U^3 - 3U) + \frac{E_x(X)}{24} \cdot Z_u \cdot (U^4 - 6U^2 + 3)\right]$$

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad U = \frac{x-m}{\sigma}, \quad Z_u = \frac{1}{\sqrt{2\pi}} e^{-\frac{U^2}{2}},$
$s_k(x)$ - asymmetry, $E_x(X)$-exess
Let (E, S) – be a measurable spase. Let

$$\mu(A) = \int_A f_{Sh}(x)dx, \quad A\epsilon L(R)$$

Probability Sharle given on $(R, L(R))$, vohere $f_{sh}(x)$ be spectral density Sharle and $L(R)$ Lebesgue σ - algebra in R. Let $\{\mu_i, i\epsilon I\}$ the corresponding Sharle probability measures.

Definition 1.2. An Object $\{E, S, \mu_h, h\epsilon H\}$ is called a Sharle statistical structure.

Definition 1.3. A Sharle statistical structure $\{E, S, \mu_h, h\epsilon H\}$ is called orthogonal (singular) if a family of Sharle probability measures $\{\mu_i, i\epsilon I\}$ constricts of pairwise singular measures $(i,e.\ \mu_{h'} \perp \mu_{h''} \ \ \forall h' \neq h'')$.

Definition 1.4. Sharle statistical structure $\{E, S, \mu_h, h\epsilon H\}$ is called weakly separable if then exists a family of S-measurable sets $\{ X_h, i\epsilon I\}$ Such that the relations are fulfilled:

$$\mu_h(X_{h'}) = \begin{cases} 1, & if\ h = h' \\ 0, & if\ h \neq h' \end{cases}$$

Definition 1.5. A Sharle statistical structure $\{E, S, \mu_h, h \epsilon H\}$ Is called separable if there exists a family of S-measurable sets $\{ X_h, h\epsilon H\}$ Such that the relations are fulfilled:

1. $\mu_h(X_{h'}) = \begin{cases} 1, & if\ h = h' \\ 0, & if\ h \neq h' \end{cases}$
2. $\forall h,\ h' \in H\ \ card(X_h \cap X_{h'}) < c\ \ if\ h \neq h'$

Where c denotes the continuum power

Definition 1.6 A Sharle statistical structure $\{E, S, \mu_h, h\epsilon H\}$ is called strongly separable if there exists a disjoint family of S-measurable sets $\{ X_h, h\epsilon H\}$ such that the relations are fulfilled:

$$\mu_h(X_h) = 1, \forall i \in I.$$

Remark 1.1 from strong separability there follows separability there follows orthogonality but not vice versa.

Example 1.1 let *E*=*RxR* (where $R = (-\infty, +\infty)$ and *L (RxR)* be a lebesgue σ- algebra of subsets of *RxR*. As a set of hypotheses consider the set *H*=*R*. let $X_h = \{(-\infty < x < +\infty), y = h, h \in R\}$. And let

$$\mu(A) = \int\limits_A f_{Sh}(x)dx$$

Be the Sharle linear measures on $X_h, h\epsilon R$

The Sharke statistical structure $\{R \times R, L(R \times R), \mu_h, h \in R\}$ is continuum strongly separable statistical structure.

Let H be the set of hypotheses and let $B(H)$ be σ-algebra of subsets of H which contains all finite subsets of H.

Definition 1.7. we will say that the statistical structure $\{E, S, \mu_h, h\epsilon H\}$ admits a consistent criterium for hypotheses testing if there exist at least one measurable mapping

$$\delta: (E, S) \to \big(H, B(H)\big),$$

Such that

$$\mu_h(\{x: \delta(x) = h\}) = 1,\ \ \forall h \in H.$$

The notion and corresponding construction of consistent criteria for hypotheses testing was introduced and studied by Z. Zerakidze (see [5]).

Remark 1.2. if the Sharle statistical structure $\{E, S, \mu_h, h\epsilon H\}$ admits a consistent criterion for hypothesis testing, then the Sharle statistical structure $\{E, S, \mu_h, h\epsilon H\}$ is strongly separable but not vice versa.

(see example 1.1).

Example 1.2. let $E = R \times R \times R = R^3$, let S be a Borel σ-algebra on R^3. let take S-measurable sets

$$X_h = \begin{cases} -\infty < x < +\infty, \quad -\infty < y < +\infty, \quad z = h\ if\ h \leq [0,1]; \\ x = h - 2, -\infty < y < +\infty, -\infty < z < +\infty, if\ h \in [2,3]; \\ -\infty < x < +\infty, \qquad y = h - 4, -\infty < z < +\infty, if\ h \in [4,5]; \end{cases}$$

and assume that μ_h are plane Sharle measures on X_h Then the Sharle statistical structure $\{R^3, S, \mu_h, h \in [0,1] \cup [2,3] \cup [4,5]\}$ is weakly separable, but not strongly separable.

Example 1.3 let $E=[0,1]\times[0,1]$, let $B=[0,1]\times[0,1]$, be a Borel σ-algebra of subsets of E. As a set of hypotheses consider the set $H=[0,1]\cup[2,3]$ let us take the $B=[0,1]\times[0,1]-$ measurable sets

$$X_h=\begin{cases}0\le x\le 1, \quad y=h, \quad if\ h\in[0,1];\\ x=h-2, 0\le y\le 1, \ \ if\ h\in[2,3];\end{cases}$$

and denote by $\mu_h, h\in[0,1]\cup[2,3]$ linear, normed, Sharle measures on X_h. Then the Sharle statistical structure $\{[0,1]\times[0,1], B([0,1]\times[0,1]), \mu_h, h\in[0,1]\cup[2,3]\}$, is a separable statistical structure. Suppose that it admits a consistent criterium for hypotheses testing

$$\delta:([0,1]\times[0,1], B([0,1]\times[0,1]))\to(H, B(H)),$$

with

$$\mu_h(\{x:\delta(x)=h\})=1, \ \forall h\in[0,1]\cup[2,3].$$

Let's introduce sets $A_1=\{x:\delta(x)\in[0,1]\}$ and $A_2=\{x:\delta(x)\in[2,3]\}$ it is clear that A_1 and A_2 $B([0,1]\times[0,1])$ measurable Sets and we have

$\mu_h(A_1\cap\{[0,1]\times\{h\})=1\ \ \forall h\in[0,1]$ and

$\mu_h(A_2\cap\{h-2\}\times[0,1])=1\ \ \forall h\in[2,3]$

Further, according to the Fubin theorem we conclude that $\mu(A_1)=1$ and $\mu(A_2)=1$ (where μ is the Sharle plane measure).

From here, taking into account that $A_1\cap A_2=\emptyset$ and $A_1\cup A_2=[0,1]\times[0,1]$ we verify that $\mu([0,1]\times[0,1])=2$ which contradicts rhe fact that $\mu([0,1]\times[0,1])=1$. Hence, the Sharle separable statistical structure does not admit a consistent criterium for hypotheses testing.

Theorem 1.1 let $\{E,S,\mu_{h_i}, i\epsilon N\}$ $(N=1,2,\dots,n,\dots.)$ Be on orthogonal Sharle statistical structure, then this statistical structure admit a consistent criterium for hypotheses testing.

Proof due to the singulatito of Sharle statistical structure $\{E,S,\mu_h, n\epsilon N\}$ there exists the family of S-measurable sets $\{X_{ik}\}$ such that for any $i\neq k$: | $\mu_{h_k}(X_{i_k})=0$ and $\mu_{h_i}(E-X_{ik})=0$, Therefore if consider the sets

$$X_i=\bigcup_{k\neq i}(E-X_{ik}),$$

we get $\mu_{h_i}(X_i)=0$, Hence, $\mu_{h_i}(E-X_i)=1$. On the other hand, for $k\neq i$ we have $\mu_k(E-X_i)=0$, It means that the Sharle statistical structure $\{E,S,\mu_{h_i}, i\in N\}$ is weakly seperable. Therefore, there exists the family of S-measurable sets $\{X_{h_i}\ i\in N\}$ such, that

$$\mu_{h_i}(X_{h_i})=\begin{cases}1, & \text{if } i=j\\ 0, & \text{if } i\neq j\end{cases}$$

Consider now the sets

$$\bar{X}_{h_i}=X_{h_i}-\left(X_{h_i}\bigcap\left(\bigcup_{k\neq 0}X_{h_k}\right)\right), i\in N$$

It is obvious that these sets are S-measurable disjocht sets and

$$\mu_{h_i}(\bar{X}_{h_i}) = 1,\ \forall i \in N$$

Wet us define the mapping

$$\delta: (E, S) \to (H, B(H))$$

in the following way: $\delta(\bar{X}_i) = h_i, \forall i \in N$.

Then we have

$\{x: \delta(x) = h_i\} = \bar{x}_i$

and

$\mu_{h_i}(\bar{X}_{h_i}) = \mu_{h_i}(\{x: \delta(x) = h_i\}) = 1,\ \forall i \in N$

Hence δ is a consistent viterion for hypo theses testing.

2. The consistent oriterion for hypotheses testing of Sharle strongly separgble statistical stractares.

Let $\{\mu_{\mathrm{h}}, h \in H\}$ be Sharle prebabality measares defined on the measurable space (E, S), For each $h \in \mathrm{H}$ denote by $\bar{\mu}_{\mathrm{h}}$ the completion of the measure μ_{h} and denote by $\mathrm{dom}(\bar{\mu}_{\mathrm{h}})$ the σ-algebra of all $\bar{\mu}_{\mathrm{h}} -$ measurable sabsets of E. Let

$$S_1 = \bigcap_{h \epsilon H} \mathrm{dom}(\bar{\mu}_{\mathrm{h}}) \ .$$

Definition 2.1. A Sharle statistical structure $\{E, S_1, \bar{\mu}_{\mathrm{h}}, h \epsilon H\}$ is called strongly separable if there exists the family of S_1 -measurable sets $\{z_h, h \epsilon H\}$ such that the relations are fulfilled:

1 $\bar{\mu}_h(z_h) = 1, \forall h \in H;$

2 $Z_{h_1} \cap z_{h_n} = \emptyset\ \forall h_1 \neq h_2;$

3 $\bigcup_{h \epsilon H} z_h = E.$

Definiteon 2.2. We will say that the orthogonal Sharle statistical structure admits a consistent criterion for testing hypothesis if there exists at least one measurathe mapping

$$\delta: (E, S_1) \to (H_1 B(H)) \text{ , such that}$$

$$\bar{\mu}_h(\{x: \delta(x) = h\}) = 1,\ \ \forall h \in H.$$

Theorem 2.1 in order that the Sharle statistical structure $\{E, S_1, \bar{\mu}_{\mathrm{h}}, h \epsilon H\}$, $\mathrm{card} H = c$ admitted a consistent criterion for hypotheses testing it is necessary and sufficient that this statistical structure was strongly separable (see definition 2.1).

Proof. Necessity. The existence of a consistent criterion for hypothesis testing (see definition 2.2) means that there exist at least one measurable mapping

$$\delta:(E,S_1)\to(H,B(H))$$

Such that

$$\bar{\mu}_h(\{x:\delta(x)=h\})=1,\forall h\epsilon H.$$

Denoting $z_h=\{x:\delta(x)=h\}$ for $h\epsilon H$ we get:

1) $\bar{\mu}_h(z_h)=\bar{\mu}_h(\{x:\delta(x)=h\})=1,\forall h\epsilon H;$
2) $z_{h_1}\cap z_{h_2}=\emptyset \quad \{x:\delta(x)=h_1\}\cap\{x:\delta(x)=h_2\} \quad \forall h_1\neq h_2, \quad h_1,h_2\epsilon H;$
3) $\bigcup_{h\in H} Z_h=\{x:\delta(x)\epsilon H\}=E$

Sufficiency, since the Sharle statistical structure $\{E,S_1,\bar{\mu}_{\mathrm{h}},h\epsilon H\}$ Is strongly separable (see definition 2.1.) there exist a family $\{z_{\mathrm{h}},h\epsilon H\}$ of elements of the σ-algebra

$$S_1=\bigcap_{h\epsilon H} dom(\bar{\mu}_h).$$

Such that

1) $\bar{\mu}_h(z_h)=1,\forall h\epsilon H;$
2) $z_{h_1}\cap z_{h_2}=\emptyset \quad \forall h_1\neq h_2;$
3) $\bigcup_{h\in H} Z_h=E$

For $x\epsilon E$ we put

$$\delta(x)=h$$

Where h is a unique hypothesis from the H for which $x\epsilon Z_h$ the existence and uniqueness of such hypothesis h can be proved using conditions 2) and 3)

Take now $y\epsilon B(H)$. Then

$$\{x:\delta(x)\epsilon y\}\epsilon\bigcup_{h\in y} Z_h$$

We have to show that

$$\{x:\delta(x)\epsilon y\}\epsilon dom(\bar{\mu}_{h_0})$$

For each $h_0\epsilon H$

If $h_0\epsilon y$ then

$$\{x:\delta(x)\epsilon y\}\epsilon\bigcup_{h\in y} Z_h=Z_{h_0}\bigcup\left(\bigcup_{h\in y-\{h_0\}} Z_h\right)$$

On the one hand the conditions 1), 2) and 3) follows that

$$Z_{h_0}\epsilon S_1$$

On the other hand the inclusion

$$\bigcup_{h\in y-\{h_0\}} Z_h\subseteq(E-Z_0)$$

Implies that

$$\bar{\mu}_{h_0}\left(\bigcup_{h\in y-\{h_0\}} Z_h\right) \le \bar{\mu}_{h_0}(E - Z_0) = 0$$

And hence

$$\bar{\mu}_{h_0}\left(\bigcup_{h\in y-\{h_0\}} Z_h\right) = 0.$$

And hence

$$\bigcup_{h\in y-\{h_0\}} Z_h \in dom\big(\bar{\mu}_{h_0}\big).$$

Since $dom\big(\bar{\mu}_{h_0}\big)$ Is σ-algebra, we conclude that

$$\{x: \delta(x)\epsilon y\} = Z_{h_0} \cup \left(\bigcup_{h\in y-\{h_0\}} Z_h\right) \in dom\big(\bar{\mu}_{h_0}\big).$$

If $h_0 \notin y$ Then

$$\{x: \delta(x)\epsilon y\}\epsilon \bigcup_{h\in y} Z_h \subseteq (E - Z_{h_0})$$

And we conclude that

$$\bar{\mu}_h(\{x: \delta(x) \in y\}) = 0.$$

The last relation implies that

$$\{x: \delta(x)\epsilon y\}\epsilon dom\big(\bar{\mu}_{h_0}\big), \forall y \in B(H).$$

Thus we have shown the validity of the relation

$$\{x: \delta(x)\epsilon y\}\epsilon dom\big(\bar{\mu}_{h_0}\big)$$

For an arbitrary $h_0 \in H$ Hence,

$$\{x: \delta(x)\epsilon y\}\epsilon \bigcap_{h\in H} dom(\bar{\mu}_h) = S_1$$

We have shown that the map

$$\delta: (E, S_L) \to (H, B(H))$$

Is measurable map

Since $B(H)$ contains all singletons of H, we ascertain that $\bar{\mu}_h(\{x: \delta(x) = h\}) = \bar{\mu}_h(z_h) = 1, \forall h \in H.$

References

[1] Borovkov A.A. Mathematics statistics. Estimation of parameters. Testing of hypotheses. “Nauka”, Moscow, 1984, English transl. mathematical statistics. Gordon. Breach. Amsterdam 1998.

[2] Kharazishvili A. Topological aspects of measure theory (in Russian) “Naukova Dumka”, Kiev, 1984.

[3] Kharazishvili A. Nonmeasureable Sets and Functions. North-Holland Mathematics Studies. Elsevier, Amsterdam, 2004.
[4] Zerakidze Z. On weakly divisible and divisible families of probability measures (in Russian). Bull. Acad. Sci. Georg. SSR 113(2), 1984, pp. 273-275.
[5] Zerakidze Z. Generalization criteria of Neiman-Pearson. Georgian Technical University. Proceedings of the International Scientifics Conference on "Information Technologies", Tbilisi, 2008, pp. 22-24.
[6] Zerakidze Z., Purtokhia O. The weakly consistent, strongly consistent and consistent estimates of the parameters. Reports of Enlarged Sessions of the I. Vekua Institute of Applied Mathematics, 31, 2017, pp. 151-154.
[7] Danko P. Ye., Popov A.G., Kozhevnikova T.Ya., Danko S. P. Vysshaya matematika v uprazhneniyakh i zadachakh, chast' 2. «Mir i obrazovaniye», Moskva, 2009.

შარლეს სტატისტიკური სტრუქტურები

ზ. ზერაკიძე, ჯ. ქირია, თ. ქირია

რეზიუმე

სტატიაში აგებულია შარლეს ძლიერად და სუსტად განცალებადი სტრუქტურები, რომელთათვისაც არ არსებობს ჰიპოთეზათა შემოწმების ძალდებული კრიტერიუმი. ამავე სტატიაში განხილულია უფრო ფართე σ-ალგებრა და ახალი განმარტება ძლიერად განცალებადი სტატისტიკური სტრუქტურები, რომელებისათვის ყოველთვის არსებობს ჰიპოთეზა შემოწმების ძალდებული კრიტერიუმი.

Статистические структуры Шарлье

З. С. Зеракидзе, Дж. К. Кириа, Т. В. Кириа

Резюме

В статье построены сильно и слабо разделимые статистические структуры Шарлье, для которых не существует состоятельных критериев для проверки гипотез. Далее строится более широкая σ −алгебра и по новому определены сильно разделимые статистические структуры Шарлье, для которых всегда существуют состоятельные критерии для проверки гипотез.