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The Consistent Criteria for Hypotheses Testing for Sharle Statistical Structure

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ABSTRACT

In this paper, we define Sharle statistical Structure such that the probability of any kind of error is zero for given criterion. The necessary and sufficient conditions for the existence of such criteria are given.

Key words: *Consistent criterion, statistical structures*.

I. Introduction

Statistics of random processes is used in various fields of science and technology (for example, in theoretical physics, genetic, economics, radio physics, geophysics).

When using random processes as models of real phenomena, the question of determining the probabilistic characteristics of the processes arises. To determine these characteristics should be use statistical methods. Among the problems of statistics, a class of problems is distinguished in which the number of observations is unique.

Despite the uniqueness of observation, is many cases, one can authentically determine the values of unknown distribution parameters or reliably choose one of an infinite number of competing hypotheses about the exact form of the distribution. In the case when a parameter or hypothesis is determined by one observation reliably, it is said that for it there exists a consistent estimate of parameter or a consistent criterion for hypothesis testing. This article is devoted to the question a consistent criterion for hypothesis testing for Sharle statistical structure.

It is said that an error of the h-th kind of the δ criterion accors, if the criterion rejects the main hypothesis of Ha. The following probability $\{\alpha_h(\delta) = \mu_h(\{x: \delta(r) \neq h\})$ Is called the probability of an erroe of the h-th kind for a given criterion δ. Example 1 nd 2 show a general trend-when we decrease one of the probabilities of an error, the other, as a rule, increases.

Example 1.1 Consider the case when there are two simple hypheses h_1 and h_2 about the distribution of population and criterion $\delta: R^n \to \{h_1, h_2\}$ such that $\delta(x) \equiv h_1$ then the probability of an error of the first kind is zero $\alpha_1 = \mu_{h_1}(\{x: \delta(r) \neq h_1\}) = 0$ and the probability of an error of the second kind is equal to one $\alpha_2 = \mu_{h_2}(\{x: \delta(r) \neq h_2\}) = 1$.

Example 1.2. There are observations from the normal distribution in R with variation one and different means $a \in R$ And two simple hypotheses $H_1 = \{a = 0\}$ And $H_2 = \{a = 1\}$ consider the following criteria

$$\delta(x) = \begin{cases} H_1 \, if \ \ x \le c; \\ H_2 \, if \ \ x \le c, \end{cases}$$

From $c \in R$. It obvious that with the increase of the number c the probability of an error of the first type decreases, and the probability of an error of the second kind increases.

2. The consistent criteria for hypotheses testing of Sharle statistical structure let (E,S) – be a measurable space. The following definitions are taken from ([1] – [7])

Definition 2.1. we will say that X random value is the Sharle (see [7]) distribution if this density given by formul

$$f_{sh} = f(x) + \frac{1}{\sigma}\left[\frac{S_k(x)}{6} \cdot Z_u \cdot (u^3 - 3u) + \frac{E_x(x)}{24} Z_u \cdot (u^4 - 6u^2 + 3)\right] \quad (1)$$

$$f_x = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-m)^2}{2\sigma^2}} \qquad u = \frac{x-m}{\sigma} \quad Z_u = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{u^2}{2}}$$

$S_k(x)$ - asymmetry, $E_x(x)$ - excess

Let

$$\mu(A) = \int_A f_{sh}(x)dx, \qquad A \epsilon L(R)$$

Probability Sharle given on $(R, L(R))$ where $f_{sh}(x)$ Be spectral density Sharle and $L(R)$lebesgue σ −algebra in R.

Let $\{\mu_{i,} \ i\epsilon I\}$ the corresponding Sharle probability measures .

Definition 2.2. an object $\{E, S, \mu_i, \ i\epsilon I\}$ is called a Sharle statistical structure.

Definition 2.3. a Sharle statistical structure $\{E, S, \mu_i, \ i\epsilon I\}$ Is called orthogonal (singular) if a family of Sharle probability measures $\{\mu_{i,} \ i\epsilon I\}$ Consists of pairwise singular measures (i.e. $\mu_i \perp \mu_j, \ \forall \, i \neq j$).

Example 2.1 let $E = [0, 1]$ S be a Borel σ −algebra of subsets of $[0, 1]$.

Let $\mu_1(B) = 2l\left(B \cap \left[0, \frac{1}{2}\right]\right)$, $\mu_2(B) = 2l\left(B \cap \left[\frac{1}{2}, \ 1\right]\right)$ and $\mu_3(B) = 3l\left(B \cap \left[0, \ \frac{1}{3}\right]\right)$ BCS., where L lebesgue measure on S. then $\mu_1 \perp \mu_2$ And $\mu_1 \perp \mu_3$ But $\mu_2 \perp \mu_3$ is not orthogonal to μ_3.

Definition 2.4. A Sharle statistical structure $\{E, S, \mu_i, \ i\epsilon I\}$ is called weakly separable if there exists a family of S-measurable sets $\{X_i, \ \ i \in I\}$ such that the relations are fulfilled:

$$\mu_i(X_j) = \begin{cases} 1, & if \ \ i \ = j \\ 0, & if \ \ i \ \neq \ j \end{cases}$$

Definition 2.5 A Sharle statistical structure $\{E, S, \mu_i, \ i\epsilon I\}$ Is called separable if there exists a family of S-measurable sets $\{X_i, \ \ i \in I\}$ Such that the relations are fulfilled:

1) $\mu_i(X_j) = \begin{cases} 1, & if \ i = j \\ 0, & if \ i \neq j \end{cases}$

2) $\forall \ i,j \in I \quad card\ (X_i \cap X_i) < c$ if $i \neq j,$

Where c denotes the continuum power.

Definition 2.6. A Sharle statistical structure $\{E, S, \mu_i, \ i\epsilon I\}$ is called strongly separable if there exists a disjoint family of S-measurable sets $\{X_i, \ i \in I\}$ such that the relations are fulfilled: $\mu_i(X_i) = 1, \ \forall \ i \in I$

Remark 2.1. From strong separability there follows separability there follows weak separability and from weak separability there follows orthogonality but not vice versa.

Remark 2.2. On an arbitrary set E of continuum power one can define a Sharle orthogonal statistical structure having the maximal possible power equal to 2^{2^c} and Sharle weakly separable statistical structure having the maximal possible power equal to 2^c, and a Sharle strongly statistical structure with the maximal possible power equal c, where c is continuum power.

Lemma 2.1. If a Sharle statistical structure $\{E, S, \mu_i, \ i\epsilon I\}$ Is separable then it is orthogonal.

Proof. If a Sharle statistical structure $\{E, S, \mu_i, \ i\epsilon I\}$ is separable, then there exists S-measurable sets $\{X_i, \ i \in I\}$ Such that

$$\mu_i(X_j) = \begin{cases} 1, & if \ i = j \\ 0, & if \ i \neq j \end{cases} \quad i,j \in I$$

Since $\mu_i(X_i) = 1, \ \forall \ i \in I$ and $\mu_j(X_i) = 0, \ \forall \ i \in I$ We have $\mu_i(E - X_i) = 0.$ Hence, the measurable μ_i and μ_j Are orthogonal.

The notion and corresponding construction of consistent criteria for hypotheses testing was introduced and studied by Z.Zerakidze (see [6]).

Definition 2.7 We consider the concept of the hypothesis as any assumption that determines the form of the distribution on population.

Definition 2.8. A statical criterion is any measurable mapping $\delta\ (E, S) \to (H, B(H))$ where H be the set of hypotheses and $B(H)$ be σ-algebra of subsets of H which contains all finite subsets of H.

Definition 2.9. We will say that the Sharle statistical structure $\{E, S, \mu_h, \ h\epsilon I\}$ admits a consistent criterion for hypothesis testing if there exists at least one measurable mapping

$$\delta\ (E, S) \to (H, B(H))$$

Such that

$$\mu_h: (x: \delta\ (x) = h\ ) = 1 \quad \forall \ h \in H$$

Definition 3.0. the probability

$$\alpha_h(\ \delta) = \ \mu_h: (x: \delta\ (x) \ \neq h\ )$$

Is called the probability of error of the h-th type for a given criterion….

Definition 3.1 we will say that the Sharle statistical structure $\{E, S, \mu_i, \ i\epsilon I\}$ admits a consistent criterion for hypothesis testing of any parametric function is for any real bounded measurable function

$$g: \ (H, B(H)) \to (R, B(R))$$

There exists at least one measurable function

$$f : (E, S) \to (R, B(R))$$

Such that

$$\mu_h: (x: f\ (x) = g(h)\ ) = 1 \quad \forall\ h\ \in H$$

Definition 3.1. We will say that the Sharle statistical structure $\{E, S, \mu_i,\ i\epsilon I\}$ admits an unbiased criterion for hypothesis testing if for any real bounded function

$$g : (H, B(H)) \to (R, B(R))$$

There exists at least one measurable function

$$f : (E, S) \to (R, B(R))$$

Such that

$$\int_E\ f\mu_h(dx) = g(x)\ ,\ \ \forall\ h\ \in H$$

Theorem 2.1. Let the Sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ Admit a consistent criterion for hypothesis testing of any parametric function, then the Sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ Is weakly separable.

Theorem 2.2. Let the Sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$Admit a consistent criterion for hypothesis testing, then the statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ Admits a consistent criterion for hypothesis testing of any parametric function, which in turn, implies the existence of an unbiased criterion for hypothesis testing.

Theorem 2.1. Since the Sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ Admits a consistent criteria of hypothesis any parametric function, denote by $f(x)$ one of the corresponding consistent criterion for indicator $I_{h'}(h)$. Hence, for the sets

$$\{x: f_h\ (x) = I_{h'}(h)\} = X_h$$

We have

$$\mu_{h'}(h) = \begin{cases} 1, & if\ \ h'\ = h \\ 0, & if\ \ h'\ \neq h \end{cases}$$

The Sharle statistical stricture $\{E, S, \mu_h,\ h\epsilon I\}$ is weakly separable.

Proof Theorem 2.2. Since Sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ admits a consistens criteria of hypothesis testing there existing measurable mapping

$$\delta : (E, S) \to (H, B(H))$$

Such that

$$\mu_h\ (x: \delta\ (x) = h\ \ ) = 1 \quad \forall\ h\ \in H$$

Let

$$g:\ (H, B(H)) \to (R, B(R))$$

Be any real, bounded, measurable function and define the function $\delta_g\ as$ follows:

$$\delta_g = g\big(\delta(x)\big)$$

Then we obtain $\{x: \delta_g = g(h)\} = \{x: \delta(x) = h\}$

$$\mu_h\ (x: \delta\ (x) = g(h)\ ) = \mu_h\ (x: \delta\ (x) = h\ ) = 1 \quad \forall\ h\ \in H \quad \text{and}$$

$$\int_E\ f\mu_h(dx) = g(x)\ ,\ \ \forall\ h\ \in H$$

Theorem 2.3. If the Sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ Admits a consistent δ criteria of hypothesis testing then this statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ is strongly separable, but not versa.

Proof. Since the Sharle statisitical stricture admits a consisten criterion of hypothesis testing there existing δ measersble maping

$$\delta : (E, S) \to (H, B(H))$$

Such that

$$\mu_h\ (x: \delta\ (x) = h\ ) = 1 \quad \forall\ h\ \in H$$

Let $X_h = \{x: \delta\ (x) = h\} = 1.$ $\quad X_h \cap X_{h'} = \emptyset,\ \ \forall\ h\ \neq h'$ and

$$\mu_h(X_h) = \mu_h(x: \delta\ (x) = h\ ) = 1 \quad \forall\ h\ \in H$$

Theorem 2.4. The Sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ admit a consistent criteria for hypothesis testing if and only if the probability of error of any kind is equal to zero for the criterion δ.

Proof. Necessity since the sharle statistical structure $\{E, S, \mu_h,\ h\epsilon I\}$ admit a consistent criterion for hypothesis testing, there exists a measurable maping

$$\delta : (E, S) \to (H, B(H))$$

Such that

$$\mu_h\{x: \delta\ (x) = h\,\} = 1 \quad \forall\ h\ \in H$$

There for

$$\alpha_h(\ \delta) =\ \mu_h: (x: \delta\ (x)\ \neq h\ ) = 0\ , \quad \forall\ h\ \in H$$

Sufficiency since thes probability of error of any kind is equal to zero, we have

$$\alpha_k(\ \delta) =\ \mu_h: (x: \delta\ (x)\ \neq h\ ) = 0\ , \quad \forall\ h\ \in H$$

On the ather hand

$$\{x: (\delta(x) = h) \cup (x: \delta(x) \neq h)\} = E$$

and

$$\mu_h[(x: (\delta(x) = h) \cup (x: \delta(x) \neq h)] = \mu_h(E) = 1$$

and

$$\mu_h[(x: \delta(x) = h) \cup (x: \delta(x) \neq h)] =$$
$$=\mu_h(x: \delta(x) = h) + \mu_h(x: \delta(x) \neq h)] = \mu_h(x: (\delta(x) = h) = 1, \ \forall h \in H$$

Example 2.1. let $E = R \times R$ (where $R = (-\infty, +\infty)$ and $B(R \times R)$ be a Borel σ-algebra of subsets of $R \times R$. As a set of hypotheses consider the se $H = Q^+$, where Q^+ is the set of positive rational nambers.

Let $X_h = \{-\infty < x < +\infty, y = h, h \in Q^+\}$

And let

$$\mu_h(A) = \int_A f_{sh}(x)dx$$

be the Sharle linear measures on X_h, $h \in Q^+$.

The Sharle statistical structure $X_h = \{R \times R, \mathrm{B}(R \times R), \mu_h, h \in Q^+\}$ is countable Sharle strongly separabele statistical structure.

If we now define the mapping

$$\delta: (R \times R, B(R \times R) \rightarrow (Q^+, B(Q^+))$$

By the formula

$$\delta(X_h) = h, \qquad h \in Q^+$$

We get

$$\mu_h(\{x: \delta(x) = h\}) = 1, \qquad \forall h \in Q^+$$

Thus, this Sharle statistical structure $\{R \times R, B(R \times R), \forall h \in Q^+\}$ admits a consistent criterion for hypothesis testing.

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ჰიპოთეზათა შემოწმების ძალდებული კრიტერიუმები შარლეს სტატისტიკური სტრუქტურებისათვის

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რეზიუმე

სტატიაში განმარტებულია შარლეს სტატისტიკური სტრუქტურები. აგებულია ისეთი შარლეს სტატისტიკური სტრუქტურა, რომლისთვისაც ჰიპოთეზათა შემოწმების ძალდებული კრიტერიუმისათვის ყველა რიგის შეცდომის ალბათობა ნულის ტოლია.

Состоятельные критерии для проверки гипотез статистических структур Шарлье

З. С. Зеракидзе, Дж. К. Кириа, Т. В. Кириа

Резюме

В этой статье мы определяем статистическую структуру Шарлье так, что вероятность любого вида ошибки равна нулю для данного критерия. Приведены необходимые и достаточные условия существования таких критериев.