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On the effective numerical methods of solution of shallow water problem. Realization of the model for the easternmost part of the Black Sea

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Abstract

Two versions of the numerical method of solution of a shallow water equation system based on the two-cycle splitting method with realization for the easternmost water area of the Black Sea are presented. In the first version solution of the equation system, which is received as a result of splitting of the basic equation system with respect to physical processes (adaptation stage), is carried out with use of factorization method regarding to current velocity components. In the second version the equation system received on the adaptation stage is reduced to the oscillation equations of string, which are also solved by factorization method. The algorithms considered in the present paper do not require use of iterative methods, which considerably increase machine time for realization of a task.

1. Statement of the problem

In a 3-D area Ω $\{(x,y,z):(x,y)\in\Omega_0(t),\ -\zeta(x,y,z)\le z\le h(x,y)\}$ limited from above by the free surface $\zeta(x,y,z)$, from below - by the motionless bottom $h(x,y)$ and by the enough smooth lateral surface σ, let's consider a mathematical task describing dynamics of shallow water area. The area Ω represents coastal part of the deep sea or shallow lake with depth H=$-\zeta(x,y,z)$+$h(x,y)$. The boundary σ_0 of 2-D area Ω_0 is formed by crossing of the free surface with the bottom relief. The coordinate system is located so that the plane xoy coincides with undisturbed water surface, z is directed vertically downwards. By taken into consideration mobility of the boundary of the considered area the problem describes processes of drainage and flooding because of fluctuations of a see surface level. In other words, the problem consists in definition of the free surface level ζ and current characteristics averaged on a vertical

$$\bar{u}=\frac{1}{H}\int_{-\zeta}^{h}u\,dz \qquad \text{and} \qquad \bar{v}=\frac{1}{H}\int_{-\zeta}^{h}v\,dz\,,$$

where u and v are horizontal components of the current velocity vector on coordinates x and y, respectively.

With the purpose of reception of a shallow water problem the equations of movement and continuity for an incompressible liquid are considered, integration of which on a vertical from z = $-\zeta$ to z=$h(x,y)$ with use of appropriate transformations enables us to receive the system of differential equations of the shallow water theory [1-5]:

$$\frac{\partial U}{\partial t}+\frac{\partial uU}{\partial x}+\frac{\partial vU}{\partial y}-lV+gH\frac{\partial\zeta}{\partial x}=\nabla\mu\nabla U+f_1,$$

$$\frac{\partial V}{\partial t}+\frac{\partial uV}{\partial x}+\frac{\partial vV}{\partial y}+lU+gH\frac{\partial\zeta}{\partial y}=\nabla\mu\nabla V+f_2, \qquad (1)$$

$$\frac{\partial \zeta}{\partial t}+\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0,$$

where $U=\bar{u}H$, $V=\bar{v}H$, $\nabla\mu\nabla=\nabla_x\mu\nabla_x+\nabla_y\mu\nabla_y$,

$$f_1=\frac{1}{\rho}(\tau_{xz}^0-\tau_{xz}^1)-\frac{gH}{\rho}\frac{\partial P_a}{\partial x},\qquad f_2=\frac{1}{\rho}(\tau_{yz}^0-\tau_{yz}^1)-\frac{gH}{\rho}\frac{\partial P_a}{\partial y}.$$

For a pressure of sea water the hydrostatic law is accepted

$$P=g\rho(\zeta+z)+P_a.$$

Here P_a is the atmospheric pressure above the free sea surface, g is the gravity acceleration, μ is the factor of turbulent viscosity, the density $\rho=const$, τ_{xz}^0 and τ_{yz}^0 are wind stress components on the free surface $z=-\zeta$, τ_{xz}^1 and τ_{yz}^1 are the bottom friction components on $z=h$.

The system of equations (1) is solved under the following boundary and initial conditions:

$$U=\tilde{U}(x,y,t),\qquad V=\tilde{V}(x,y,t)\quad \text{on}\quad \sigma_2, \tag{2}$$

$$U=0,\qquad V=0\quad \text{on}\quad \sigma_1, \tag{3}$$

$$U=U^0(x,y),\quad V=V^0(x,y),\quad \zeta=\zeta^0(x,y)\quad \text{if}\quad t=0, \tag{4}$$

where σ_1 is the lateral solid surface adjacent to the land, σ_2 is the liquid (open) boundary separating the sea basin from the remaining water area. On the open boundary σ_2 the vector of a complete flow is the given function of horizontal coordinates and time.

We assume that the input data of the given task have sufficient smoothness providing solvability of the task (1) - (4) [1, 4, 6, 7].

2. Numerical method of solution

2.1. Splitting of the problem with respect to physical processes

For numerical solution of the problem (1)-(4) the entire time interval (O, T), on which the solution is searched, is broken up into equal intervals $t_{j-1}\le t\le t_{j+1}$ and is supposed that on the each interval the components of the vector velocity $\bar{\bar{u}}$ are known from the previous time step [8, 9, 10].

Let's consider vectors φ, Q, F, and matrixes A and B

$$\varphi=\begin{Vmatrix}U\\V\\\zeta\end{Vmatrix},\quad Q=\begin{Vmatrix}f_1\\f_2\\0\end{Vmatrix},\quad F=\begin{Vmatrix}U^0\\V^0\\\zeta^0\end{Vmatrix},\quad A=\begin{Vmatrix}\Pi & -l & gH\frac{\partial}{\partial x}\\ l & \Pi & gH\frac{\partial}{\partial y}\\ gH\frac{\partial}{\partial x} & gH\frac{\partial}{\partial y} & 0\end{Vmatrix},\quad B=\begin{Vmatrix}1&0&0\\0&1&0\\0&0&gH\end{Vmatrix}.$$

Here $\Pi=div\bar{\bar{u}}-\nabla\mu\nabla$. Then the equation system (1) we shall write in the operator form

$$B\frac{\partial\varphi}{\partial t}+A\varphi=Q \tag{5}$$

and as the initial conditions we shall accept

$$B\varphi=BF\quad \text{if}\quad t=0. \tag{6}$$

After scalar multiplying the equation (5) by φ, we shall receive

$$\frac{1}{2}\frac{\partial}{\partial t}(B\varphi,\varphi) + (A\varphi,\varphi) = (Q,\varphi),$$

where the scalar product is determined as follows

$$(a,b) = \sum_{i=1}^{3} \iint_{\Omega} a^i b^i d\Omega .$$

Here a^i and b^i are the vector components of functions a and b.
With taken into consideration uniform conditions corresponding to (2) and (3) it is easily to check up that the following ratio takes place:

$$(A\varphi,\varphi) > 0$$

Now we shall present the operator A as the sum of two operators

$$A = A_1 + A_2 ,$$

where

$$A_1 = \begin{Vmatrix} \Pi & 0 & 0 \\ 0 & \Pi & 0 \\ 0 & 0 & 0 \end{Vmatrix}, \qquad A_2 = \begin{Vmatrix} 0 & -l & gH\frac{\partial}{\partial x} \\ l & 0 & gH\frac{\partial}{\partial y} \\ gH\frac{\partial}{\partial x} & gH\frac{\partial}{\partial y} & 0 \end{Vmatrix}$$

Similarly, it is possible to show that the following ratios take place

$$(A_1\varphi,\varphi) > 0, \qquad (A_2\varphi,\varphi) = 0 .$$

These conditions are necessary for construction of steady finite-difference schemes of splitting.

Let's now $t_j - t_{j-1} = \tau$ and on each extended time interval $t_{j-1} \le t \le t_{j+1}$ the two-cyclic method of splitting on physical processes to the task (5), (6) we shell apply. Then if we assume that φ^{j-1} is the solution of the task (5), (6) at the moment t_{j-1} then we receive the following tasks:

on the first time interval $t_{j-1} \le t \le t_j$ -

$$B\frac{\partial \varphi_1}{\partial t} + A_1\varphi_1 = Q , \quad B\varphi_1^{j-1} = B\varphi^{j-1} . \tag{7}$$

on the time interval $t_{j-1} \le t \le t_{j+1}$ -

$$B\frac{\partial \varphi_2}{\partial t} + A_2\varphi_2 = 0 , \quad B\varphi_2^{j-1} = B\varphi_1^{j}, \tag{8}$$

on the last time interval $t_j \le t \le t_{j+1}$ -

$$B\frac{\partial \varphi_3}{\partial t} + A_1\varphi_3 = Q , \quad B\varphi_3^{j} = B\varphi_2^{j+1}. \tag{9}$$

In equations (7) and (9) values of functions $\bar{u}$ и $\bar{v}$ are taken the same within each time interval $t_{j-1} \le t \le t_{j+1}$ to obtain second -order accuracy in τ [9].

In the component form the tasks (7) - (9) will accept the following form:
on the time interval $t_{j-1} \le t \le t_j$

$$\left.\begin{aligned} \frac{\partial U_1}{\partial t} + \frac{\partial u U_1}{\partial x} + \frac{\partial v U_1}{\partial y} = \nabla\mu\nabla U_1 + f_1 \;, \quad U_1^{j-1} = U^{j-1} \\ \frac{\partial V_1}{\partial t} + \frac{\partial u V_1}{\partial x} + \frac{\partial v V_1}{\partial y} = \nabla\mu\nabla V_1 + f_2 \;, \quad V_1^{j-1} = V^{j-1} \end{aligned}\right\} \tag{10}$$

with boundary conditions (2), (3) written for U_1 and V_1

$$U_1 = 0\,, \qquad V_1 = 0 \qquad \text{on} \quad \sigma_1,$$

$$U_1 = \tilde{U}, \qquad V_1 = \tilde{V} \qquad \text{on} \quad \sigma_2. \tag{11}$$

On the time interval $t_{j-1} \le t \le t_{j+1}$ we have the differential equation system

$$\left.\begin{aligned} \frac{\partial U_2}{\partial t} - lV_2 + gH\frac{\partial \zeta_2}{\partial x} = 0 \\ \frac{\partial V_2}{\partial t} + lU_2 + gH\frac{\partial \zeta_2}{\partial y} = 0 \\ \frac{\partial \zeta_2}{\partial t} + \frac{\partial U_2}{\partial x} + \frac{\partial V_2}{\partial y} = 0 \end{aligned}\right\} \tag{12}$$

at following boundary and initial conditions

$$U_{2_n} = 0 \qquad \text{on} \quad \sigma_1 \,,$$
$$\zeta_2 = \tilde{\zeta} \qquad \text{on} \quad \sigma_2 \,, \tag{13}$$

where U_{2_n} is normal component of velocity vector $\overrightarrow{U_2}$, and $\tilde{\zeta}$ is known function,

$$U_2^{j-1} = U_1^j \,, \qquad V_2^{j-1} = V_1^j \,, \qquad \zeta_2^{j-1} = \zeta^{j-1} \;; \tag{14}$$

and, on the time interval $t_j \le t \le t_{j+1}$ -

$$\left.\begin{aligned} \frac{\partial U_3}{\partial t} + \frac{\partial u U_3}{\partial x} + \frac{\partial v U_3}{\partial y} = \nabla\mu\nabla U_3 + f_1 \;, \quad U_3^j = U_2^{j+1} \\ \frac{\partial V_3}{\partial t} + \frac{\partial u V_3}{\partial x} + \frac{\partial v V_3}{\partial y} = \nabla\mu\nabla V_3 + f_2 \;, \quad V_3^j = V_2^{j+1} \end{aligned}\right\} \tag{15}$$

at the boundary conditions (2), (3) for functions U_3 and V_3 -

$$U_3 = 0 \;, \qquad V_3 = 0 \qquad \text{на} \quad \sigma_1 \,,$$
$$U_3 = \tilde{U} \;, \qquad V_3 = \tilde{V} \qquad \text{на} \quad \sigma_2. \tag{16}$$

Thus, as a result of application of the splitting method with respect to physical processes the task (1) - (4) is reduced to the consecutive solution of a set of more simple three tasks (10) - (11), (12) - (14) and (15) - (16).

2.2. Finite – difference approximation of the transfer-diffusion equation. A method of component –by-component splitting

As the differential equations (10) are same, we shall consider one equation

$$\frac{\partial \Phi_1}{\partial t} + \operatorname{div}\vec{u}\,\Phi_1 = \nabla\mu\nabla\Phi_1 + f_1, \qquad \Phi_1^{j-1} = \Phi^{j-1} \tag{17}$$

at boundary conditions (11) for function Φ_1. Here Φ_1 is any function of U_1 or V_1. An index "1" at function we shall omit for simplicity. Let

$$\mathcal{Z}_1\Phi = -\frac{\partial}{\partial x}\mu\frac{\partial \Phi}{\partial x} - \frac{\partial}{\partial y}\mu\frac{\partial \Phi}{\partial y} - \nabla\mu\nabla\Phi .$$

It is easily to be convinced that in case of homogeneous boundary conditions the differential operator is positively determined.

For the finite-difference approximation of the differential operator $\mathcal{Z}$ let's consider a difference grid, which is received at crossing of coordinate lines $x = x_k$ и $y = y_l$. As a result we receive a set of basic nodes. Let's assume that indexes k and l change in limits Ω_0: $k^0 \leq k \leq k^1$ and $l^0 \leq l \leq l^1$. In addition, it is considered that the boundary points coincide with the boundary σ_0 of the solution area Ω_0. The set of such points we shall denote by σ_{0h}. Let's consider the basic points x_k, y_l, and the auxiliary points $x_{k+1/2}$, $y_{l+1/2}$, and we shall denote

$\Delta x_{k+1/2} = x_{k+1} - x_k$, $\Delta y_{l+1/2} = y_{l+1} - y_l$ and $\Delta x_k = \frac{1}{2}(\Delta x_{k+1/2} + \Delta x_{k-1/2})$,

$\Delta y_l = \frac{1}{2}(\Delta y_{l+1/2} + \Delta y_{l-1/2})$.

Now we use the scheme of the second -order accuracy

$$\mathcal{Z}^h\Phi_{kl} = (\mathcal{Z}_0^h + \mathcal{Z}_1^h)\Phi_{kl}, \quad \mathcal{Z}_0^h = \sum_{i=1}^{2} \mathcal{Z}_{0,i}^h, \quad \mathcal{Z}_1^h = \sum_{i=1}^{2} \mathcal{Z}_{1,i}^h , \qquad (18)$$

where

$$\mathcal{Z}_{0,1}^h\Phi_{kl} = \frac{\bar{u}_{k+1/2,l}\Phi_{k+1,l} - \bar{u}_{k-1/2,l}\Phi_{k-1,l}}{2\Delta x_k} , \quad \mathcal{Z}_{0,2}^h\Phi_{kl} = \frac{\bar{v}_{k,l+1/2}\Phi_{k,l+1} - \bar{v}_{k,l-1/2}\Phi_{k,l-1}}{2\Delta y_l},$$

$$\mathcal{Z}_{1,1}^h\Phi_{kl} = -\frac{1}{\Delta x_k}\left[\frac{\mu_{k+1/2,l}}{\Delta x_{k+1/2}}\Phi_{k+1,l} - \left(\frac{\mu_{k+1/2,l}}{\Delta x_{k+1/2}} + \frac{\mu_{k-1/2,l}}{\Delta x_{k-1/2}}\right)\Phi_{kl} + \frac{\mu_{k-1/2,l}}{\Delta x_{k-1/2}}\Phi_{k-1,l}\right],$$

$$\mathcal{Z}_{1,2}^h\Phi_{kl} = -\frac{1}{\Delta y_l}\left[\frac{\mu_{k,l+1/2}}{\Delta y_{l+1/2}}\Phi_{k,l+1} - \left(\frac{\mu_{k,l+1/2}}{\Delta y_{l+1/2}} + \frac{\mu_{k,l-1/2}}{\Delta y_{l-1/2}}\right)\Phi_{kl} + \frac{\mu_{k,l-1/2}}{\Delta y_{l-1/2}}\Phi_{k,l-1}\right].$$

It is easily be convinced that the grid operators $\mathcal{Z}_{0,i}^h$ and $\mathcal{Z}_{1,i}^h$ with taken into consideration of homogeneous boundary condition are skew-symmetric and positively determined [4, 8, 9, 10]

$$(\mathcal{Z}_{0,i}^h\vec{\Phi} , \vec{\Phi}) = 0 , \quad (\mathcal{Z}_{1,i}^h\vec{\Phi} , \vec{\Phi}) > 0 , \quad i = 1,2$$

and

$$(\mathcal{Z}^h\vec{\Phi},\vec{\Phi}) > 0 , \qquad \mu_{kl} > 0.$$

Here the scalar product is determined as

$$(a, b) = \sum_{kl} a_{kl} b_{kl} \Delta x_k \Delta y_l .$$

The summation is made on all internal points of the grid area Ω_{0h}.

With taken into consideration (18), the system of finite-difference equations approximating the task (17) with the second-order accuracy on space variables accepts the following operator form:

$$\frac{d\vec{\Phi}}{dt} + \mathcal{Z}^h\vec{\Phi} = \vec{F}, \qquad (19)$$

where $\vec{\Phi}$ and $\vec{F}$ are the vector-functions with components $\{\Phi_{kl}\}$ and $\{F_{kl}\}$, respectively, given in grid points Ω_{0h}.

Now, multiplying the equation (19) scalarly by $\vec{\Phi}$, we receive

$$\frac{1}{2}\frac{d(\vec{\Phi},\vec{\Phi})}{dt} + (Z\vec{\Phi},\vec{\Phi}) = (\vec{F},\vec{\Phi}) .$$

If in (19) we assume that $\mu_{k+1/2,l+1/2} = 0$ and grid vector-function $\vec{F}$ in the right part of the difference equation, caused by boundary conditions for the equation (17), is identically equal to zero then we receive

$$\frac{d(\vec{\Phi},\vec{\Phi})}{dt} = 0$$

Thus, the norm of grid solution

$$\left\| \vec{\Phi}^j \right\| = \sqrt{\left(\vec{\Phi}^j,\vec{\Phi}^j\right)}$$

is kept, i. e.

$$\left\| \vec{\Phi}^j \right\|^2 = \left\| \vec{\Phi}^{j-1} \right\|^2 .$$

With taken into consideration the continuity equation for functions $\bar{u}_{k+1/2,l}$ and $\bar{v}_{k,l+1/2}$ the condition of conservation of substation on time takes place:

$$\frac{d(\vec{\Phi},\vec{e})}{dt} = \frac{d}{dt}\sum_{kl}\vec{\Phi}_{kl}\Delta x_k \Delta y_l = 0 ,$$

where $\vec{e}$ is the unit vector.

For approximation of the equation (19) in time on the time interval $t_{j-1} \leq t \leq t_j$ the Crank-Nicholson scheme by the subsequent application of a two-cycle splitting method is used. As a result, the problem is reduced to a sequence of more simple one-dimensional finite-difference equations which are effectively solved by a factorization method. These equations have the following form:

$$\frac{\vec{\Phi}^{j-5/6}-\vec{\Phi}^{j-1}}{\tau/2} + \left(Z_{o,1}^h + Z_{1,1}^h\right)\frac{\vec{\Phi}^{j-5/6}+\vec{\Phi}^{j-1}}{2} = 0 ,$$

$$\frac{\vec{\Phi}^{j-4/6}-\vec{\Phi}^{j-5/6}}{\tau/2} + \left(Z_{o,2}^h + Z_{1,2}^h\right)\frac{\vec{\Phi}^{j-4/6}+\vec{\Phi}^{j-5/6}}{2} = 0 ,$$

$$\frac{\vec{\Phi}^{j-2/6}-\vec{\Phi}^{j-4/6}}{\tau/2} = F ,$$

$$\frac{\vec{\Phi}^{j-1/6}-\vec{\Phi}^{j-2/6}}{\tau/2} + \left(Z_{o,2}^h + Z_{1,2}^h\right)\frac{\vec{\Phi}^{j-1/6}+\vec{\Phi}^{j-2/6}}{2} = 0 ,$$

$$\frac{\vec{\Phi}^{j}-\vec{\Phi}^{j-1/6}}{\tau/2} + \left(Z_{o,1}^h + Z_{1,1}^h\right)\frac{\vec{\Phi}^{j}+\vec{\Phi}^{j-1/6}}{2} = 0 .$$

Application of the two-cycle splitting method provides the second order accuracy in time. The received system of finite –difference equations is absolutely steady on the interval $0 \leq t \leq T$.

Thus, the considered scheme approximates the differential equation (17) with the second-order accuracy both on time and on spatial coordinates and is absolutely steady. The task (15),(16) is similarly solved.

2.3. Finite –difference approximation of equations of adaptation. Splitting of the problem

Let's consider on the time interval $t_{j-1} \le t \le t_{j+1}$ the problem of adaptation (12)-(14). index "2" at solution components we shall omit for simplicity. Here the finite –difference scheme approximates the differential problem with the second-order accuracy on spatial coordinates. The Crank-Nicholson scheme with the subsequent application of a two-cyclic method of splitting is used for approximation on time. As a whole, the scheme received here is absolutely steady and approximates (12) - (14) with the second order accuracy. Note that coefficients $\bar{u}_{k+1/2,l}$ and $\bar{v}_{k,l+1/2}$ which are in transfer-diffusion equations, are defined after solving of the adaptation problem. Before to consider the difference approximation of the system of differential equations (12) let's consider the operators A_l , A_x and A_y. We assume that

$$A_2 = A_l + A_x + A_y,$$

where

$$A_l = \begin{Vmatrix} 0 & -l & 0 \\ l & 0 & 0 \\ 0 & 0 & 0 \end{Vmatrix}, \quad A_x = \begin{Vmatrix} 0 & 0 & gH\frac{\partial}{\partial x} \\ 0 & 0 & 0 \\ gH\frac{\partial}{\partial x} & 0 & 0 \end{Vmatrix}, \quad A_y = \begin{Vmatrix} 0 & 0 & 0 \\ 0 & 0 & gH\frac{\partial}{\partial y} \\ 0 & gH\frac{\partial}{\partial y} & 0 \end{Vmatrix}.$$

Then the system of equations (12) in the operator form is

$$B\frac{\partial \varphi}{\partial t} + A_2\varphi = 0 \,. \tag{20}$$

Now if on the time interval $t_{j-1} \le t \le t_{j+1}$ we shell use the two-cycle splitting method for (20), then the problem is reduced to solution of a set of more simple problems:
on the time interval $t_{j-1} \le t \le t_j$ -

$$B\frac{\partial \varphi_1}{\partial t} + A_l\varphi_1 = 0, \qquad B\varphi_1^{j-1} = B\varphi_1^j, \qquad (\varphi_1^j \text{ is solution of (7) at moment } t = t_j),$$

$$B\frac{\partial \varphi_2}{\partial t} + A_x\varphi_2 = 0, \qquad B\varphi_2^{j-1} = B\varphi_1^j, \tag{21}$$

$$B\frac{\partial \varphi_3}{\partial t} + A_y\varphi_3 = 0, \qquad B\varphi_3^{j-1} = B\varphi_2^j$$

and on the interval $t_j \le t \le t_{j+1}$ we have

$$B\frac{\partial \varphi_4}{\partial t} + A_y\varphi_4 = 0, \qquad B\varphi_4^j = B\varphi_3^j,$$

$$B\frac{\partial \varphi_5}{\partial t} + A_x\varphi_5 = 0, \qquad B\varphi_5^j = B\varphi_4^{j+1}, \tag{22}$$

$$B\frac{\partial \varphi_6}{\partial t} + A_l\varphi_6 = 0, \qquad B\varphi_6^j = B\varphi_5^{j+1}.$$

The first equation from (21) and last equation from (22) we shall write in the component-by-component form for the basic grid points (grid points by integer indexes). As a result, we shall receive simple systems of the equations, analytical solution of which are:

$$U_{1kl}^j = U_{1kl}^{j-1}\cos lt + V_{1kl}^{j-1}\sin lt,$$

$$V^{j}_{\text{б}_{kl}} = V^{j-1}_{\text{б}_{kl}} \cos lt - U^{j-1}_{\text{б}_{kl}} \sin lt$$

and

$$U^{j+1}_{\text{б}_{kl}} = U^{j}_{\text{б}_{kl}} \cos lt + V^{j}_{\text{б}_{kl}} \sin lt,$$

$$V^{j+1}_{\text{б}_{kl}} = V^{j}_{\text{б}_{kl}} \cos lt - U^{j}_{\text{б}_{kl}} \sin lt .$$

For numerical solution of other tasks from (21),(22) let's consider their finite-difference approximation on space variables. With this purpose Let's consider approximation of the operators $\frac{\partial}{\partial x}$ и $\frac{\partial}{\partial y}$. Let the function U is given in points $x_{k^0+1/2}$ and $x_{k^1+1/2}$. Besides $U_{k^0+1/2} = a$, $U_{k^1+1/2} = b$ and $\zeta_{k^0} = c$. Then we shall use the following approximation

$$\frac{\partial U}{\partial x} \approx \nabla_k^- U \qquad \text{and} \qquad \frac{\partial \zeta}{\partial x} \approx \nabla_k^+ \zeta_k ,$$

where

$$\nabla_k^- U = \begin{cases} \dfrac{1}{\Delta x_{k^0}} U_{k^0+1/2} - \dfrac{a}{\Delta x_{k^0}} & , k = k^0 \\ \dfrac{1}{\Delta x_k}\left(U_{k+\frac{1}{2}} - U_{k-\frac{1}{2}}\right) & , k = k^0+1, k^0+2, \dots, k^1-1 \\ -\dfrac{1}{\Delta x_{k^1}} U_{k^1-1/2} + \dfrac{b}{\Delta x_{k^1}} & , k = k^1 \end{cases}$$

and

$$\nabla_k^+ \zeta_k = \begin{cases} \dfrac{1}{\Delta x_{k^0+1/2}} \zeta_{k^0+1} - \dfrac{c}{\Delta x_{k^0+1/2}} & , k = k^0 \\ \dfrac{1}{\Delta x_{k+1/2}} (\zeta_{k+1} - \zeta_k) & , k = k^0+1, k^0+2, \dots, k^1-1 \end{cases}$$

Operators ∇_l^+ and ∇_l^- are similarly determined, which approximate differential operator $\frac{\partial}{\partial y}$. Then finite-difference approximation of the differential operators A_x and A_y we will consider in the following form

$$A_x^h = \left\| \begin{matrix} 0 & 0 & gH\nabla_k^+ \\ 0 & 0 & 0 \\ gH\nabla_k^- & 0 & 0 \end{matrix} \right\|, \qquad A_y^h = \left\| \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & gH\nabla_l^+ \\ 0 & gH\nabla_l^- & 0 \end{matrix} \right\|.$$

Similarly to operators A_x and A_y the operators A_x^h and A_y^h are skew-symmetric. These conditions are necessary for construction of absolutely steady finite –difference schemes.

Taking into consideration the finite-difference operators A_x^h and A_y^h in equations (21),(22) and using the Crank-Nicholson scheme for approximation on time, after the appropriate transformations we shall receive the following systems of the finite –difference equations

$$\left.\begin{aligned} U_{2_{k+1/2,l}}^{j-1/2} + \frac{\tau g H_{k+1/2,l}}{2}\nabla_k^{+}\zeta_{2_{kl}}^{j-1/2} &= U_{2_{k+1/2,l}}^{j-1} \\ V_{2_{k,l+1/2}}^{j-1/2} &= V_{2_{k,l+1/2}}^{j-1} \\ \zeta_{2_{kl}}^{j-1/2} + \frac{\tau}{2}\nabla_k^{-}U_{2_{kl}}^{j-1/2} &= \zeta_{2_{kl}}^{j-1} \end{aligned}\right\} \qquad (23)$$

with the initial conditions

$$U_{2_{k+1/2,l}}^{j-1} = U_{1_{k+1/2,l}}^{j}, \qquad V_{2_{k,l+1/2}}^{j-1} = V_{1_{k,l+1/2}}^{j}, \qquad \zeta_{2_{kl}}^{j-1} = \zeta_{kl}^{j-1}$$

and

$$\left.\begin{aligned} U_{3_{k+1/2,l}}^{j-1/2} &= U_{3_{k+1/2,l}}^{j-1} \\ V_{3_{k,l+1/2}}^{j-1/2} + \frac{\tau g H_{k,l+1/2}}{2}\nabla_l^{+}\zeta_{3_{kl}}^{j-1/2} &= V_{3_{k,l+1/2}}^{j-1} \\ \zeta_{3_{kl}}^{j-1/2} + \frac{\tau}{2}\nabla_l^{-}V_{3_{kl}}^{j-1/2} &= \zeta_{3_{kl}}^{j-1} \end{aligned}\right\} \qquad (24)$$

with the initial conditions

$$U_{3_{k+1/2,l}}^{j-1} = U_{2_{k+1/2,l}}^{j}, \qquad V_{3_{k,l+1/2}}^{j-1} = V_{2_{k,l+1/2}}^{j}, \qquad \zeta_{3_{kl}}^{j-1} = \zeta_{2_{kl}}^{j}.$$

Here $\phi^{j-1/2} = (\phi^{j} + \phi^{j-1})/2$, ϕ is any function of U_α, V_α or ζ_α $(\alpha = 2,3)$.

Now Let's consider the system of finite-difference equations (23). Then with use of the third equation we shall exclude from the first equation function $\zeta_{2_{kl}}^{j-1/2}$, as a result we shall receive one-dimensional finite-difference equation

$$U_{2_{k+1/2,l}}^{j-1/2} - \frac{\tau^2 g H_{k+1/2,l}}{4}\nabla_k^{+}\nabla_k^{-}U_{2_{kl}}^{j-1/2} = U_{2_{k+1/2,l}}^{j-1} - \frac{\tau g H_{k+1/2,l}}{2}\nabla_k^{+}\zeta_{2_{kl}}^{j-1}. \qquad (25)$$

After salving of this equation from the third equation we shell determine the function $\zeta_{2_{kl}}^{j-1/2}$

$$\zeta_{2_{kl}}^{j-1/2} = -\frac{\tau}{2}\nabla_k^{-}U_{2_{kl}}^{j-1/2} + \zeta_{2_{kl}}^{j-1}.$$

The equation (25) is effectively solved by the factorization method.
Similarly to equation (25), from the equation system (24) for the function $V_{3_{k,l+1/2}}^{j-1/2}$, we receive the following equation

$$V_{3_{k,l+1/2}}^{j-1/2} - \frac{\tau^2 g H_{k,l+1/2}}{4}\nabla_l^{+}\nabla_l^{-}V_{3_{kl}}^{j-1/2} = V_{3_{k,l+1/2}}^{j-1} - \frac{\tau g H_{k,l+1/2}}{2}\nabla_l^{+}\zeta_{3_{kl}}^{j-1}. \qquad (26)$$

The equation (26) is salved similarly to (25).

On the time interval $t_j \le t \le t_{j+1}$, the systems of finite - difference equations (22) for grid function $U_{4_{k+1/2,l}}^{j+1/2}$, $V_{4_{k,l+1/2}}^{j+1/2}$, $\zeta_{4_{k,l}}^{j+1/2}$ and $U_{5_{k+1/2,l}}^{j+1/2}$, $V_{5_{k,l+1/2}}^{j+1/2}$, $\zeta_{5_{k,l}}^{j+1/2}$ are solved completely similarly previous systems.

Thus, the considered numerical scheme allows the solution of a shallow water problem reduce to solution of a set of more simple one-dimensional tasks which are effectively solved by the factorization method. The constructed scheme has the second-order approximation both on time and on horizontal coordinates and is absolutely steady.

At solution of the equation system (23),(24) it is possible from the continuity equation eliminate the functions $U_{2_{k+1/2,l}}^{j-1/2}$ and $V_{3_{k,l+1/2}}^{j-1/2}$, which are determined from the first and the second equations, respectively. As a result the three-point equations are received for $\zeta_{2_{k,l}}^{j-1/2}$ and $\zeta_{3_{k,l}}^{j-1/2}$, which are solved also by factorization method. Analogically are determined the grid functions $\zeta_{4_{k,l}}^{j+1/2}$ and $\zeta_{5_{k,l}}^{j+1/2}$ from (21),(22).

Now let's consider methodically other algorithm of solution of the adaptation task (the second version of the algorithm). With this purpose on the time interval $t_{j-1} \le t \le t_j$ we shall consider second and third equations from (21), and on the interval $t_j \le t \le t_{j+1}$ - the first and second equations from (22), which are approximated on spatial variable. The received equations in the component form will accept the following form:
on the time interval $t_{j-1} \le t \le t_j$ -

$$\left.\begin{array}{c} \frac{dU_2}{dt} + gH\nabla_k^+\varsigma_2 = 0 \\ \frac{dV_2}{dt} = 0 \\ \frac{d\varsigma_2}{dt} + \nabla_k^- U_2 = 0 \end{array}\right\} \tag{27}$$

with the initial conditions

$$U_2^{j-1} = U_1^j \; , \; V_2^{j-1} = V_1^j \; , \quad \varsigma_2^{j-1} = \varsigma_1^j \tag{28}$$

and

$$\left.\begin{array}{c} \frac{dU_3}{dt} = 0 \\ \frac{dV_3}{dt} + gH\nabla_k^+\varsigma_3 = 0 \\ \frac{d\varsigma_3}{dt} + \nabla_k^- V_3 = 0 \end{array}\right\} \tag{29}$$

with conditions

$$U_3^{j-1} = U_2^j, \; V_3^{j-1} = V_2^j, \quad \varsigma_3^{j-1} = \varsigma_2^j \, . \tag{30}$$

On the time interval $t_j \le t \le t_{j+1}$ we have following tasks

$$\left.\begin{array}{c} \frac{dU_4}{dt} = 0 \\ \frac{dV_4}{dt} + gH\nabla_k^+\varsigma_4 = 0 \\ \frac{d\varsigma_4}{dt} + \nabla_k^- V_4 = 0 \end{array}\right\} \tag{31}$$

with conditions

$$U_4^j = U_3^j,\ V_4^j = V_3^j, \quad \varsigma_4^j = \varsigma_3^j \tag{32}$$

and

$$\left.\begin{array}{c} \frac{dU_5}{dt} + gH\nabla_k^+\varsigma_5 = 0 \\ \frac{dV_5}{dt} = 0 \\ \frac{d\varsigma_5}{dt} + \nabla_k^- U_5 = 0 \end{array}\right\} \tag{33}$$

with initial conditions

$$U_5^j = U_4^{j+1},\ V_5^j = V_4^{j+1},\ \varsigma_5^j = \varsigma_4^{j+1}. \tag{34}$$

It should be noted that the problems (27),(28) - (33),(34) may be received from (20), if on each time interval $t_{j-1} \le t \le t_{j+1}$ we shell use the two-cycle splitting method in case, when

$$A_2 = A_x + A_y$$

Thus, the specified tasks are received from the equation system

$$\left.\begin{array}{c} \frac{dU_2}{dt} + gH\nabla_k^+\varsigma_2 = 0 \\ \frac{dV_2}{dt} + gH\nabla_l^+\varsigma_2 = 0 \\ \frac{d\varsigma_2}{dt} + \nabla_k^- U_2 + \nabla_l^- V_2 = 0 \end{array}\right\} \tag{35}$$

with conditions

$$U_2^{j-1} = U_1^j,\ V_2^{j-1} = V_1^j, \quad \varsigma_2^{j-1} = \varsigma^{j-1} \tag{36}$$

(where U_1^j and V_1^j are solutions of the task on the stage taking into account the Coriolis force and ς^{j-1} is the initial condition of the basic problem).

The problem (35),(36) may be to reduce to the oscillation equation of a membrane. After the appropriate transformations on the interval $t_{j-1} \le t \le t_{j+1}$ we shall receive the equation [8]

$$\frac{d^2\varsigma_2}{dt^2} - \nabla_k^- c^2 \nabla_k^+ \varsigma_2 - \nabla_l^- c^2 \nabla_l^+ \varsigma_2 = 0 \tag{37}$$

with initial conditions at $t = t_{j-1}$

$$\varsigma_2^{j-1} = \varsigma^{j-1} \quad \text{и} \quad \frac{d\varsigma_2}{dt} = q, \tag{38}$$

where $c = \sqrt{gH}$ и $q = \nabla_k^- U_2^{j-1} + \nabla_l^- V_2^{j-1}$.

On the other hand from each problems (27),(28) - (33),(34) analogically one-dimensional problems are received:

on the interval $t_{j-1} \le t \le t_j$ -

$$\frac{d^2\varsigma_2}{dt^2} - \nabla_k^- c^2 \nabla_k^+ \varsigma_2 = 0, \qquad \varsigma_2^{j-1} = \varsigma^{j-1}, \quad \frac{d\varsigma_2}{dt} = q,$$

$$\frac{d^2\varsigma_3}{dt^2} - \nabla_l^- c^2 \nabla_l^+ \varsigma_3 = 0\,, \quad \varsigma_3^{j-1} = \varsigma_2^j \quad , \quad \frac{d\varsigma_3}{dt} = q \tag{39}$$

and on the interval $t_j \le t \le t_{j+1}$ the following problems

$$\frac{d^2\varsigma_4}{dt^2} - \nabla_l^- c^2 \nabla_l^+ \varsigma_4 = 0\,, \quad \varsigma_4^j = \varsigma_3^j \quad , \quad \frac{d\varsigma_4}{dt} = q,$$

$$\frac{d^2\varsigma_5}{dt^2} - \nabla_k^- c^2 \nabla_k^+ \varsigma_5 = 0\,, \quad \varsigma_5^j = \varsigma_4^{j+1} \;, \quad \frac{d\varsigma_5}{dt} = q, \tag{40}$$

which as a whole give solution of the problems (27),(28) - (33),(34), i. e. of the problem (35),(36) or (36),(37).

As a result of solution of received one-dimensional problems, the grid functions ς_2^j, ς_3^j, ς_4^{j+1} and ς_5^{j+1} are determined, with help of which values of U_2^j, V_2^j, U_3^j, V_3^j, U_4^{j+1}, V_4^{j+1}, U_5^{j+1} and V_5^{j+1} are easily calculated.

As the one-dimensional tasks from (39),(40) are the same, a method of solution of the received tasks we shall consider on an example of the first task from (39).

Thus, on the time interval $t_{j-1} \le t \le t_j$ we consider the problem

$$\frac{d^2\varsigma_2}{dt^2} - \nabla_k^- c^2 \nabla_k^+ \varsigma_2 = 0\,, \tag{41}$$

$$\varsigma_2 = \varsigma^{j-1} \;, \quad \frac{d\varsigma_2}{dt} = q \qquad \text{at} \qquad t = t_{j-1}. \tag{42}$$

With the purpose of approximation on time on the interval $t_{j-1} \le t \le t_j$ of the task (41),(42) we shall consider the implicit scheme, which is similar to the Crank-Nicholson scheme [8]

$$\frac{\varsigma_{2_{kl}}^j - 2\varsigma_{2_{kl}}^{j-\frac{1}{2}} + \varsigma_{2_{kl}}^{j-1}}{\frac{\tau^2}{4}} - \nabla_k^- C_{kl}^2 \nabla_k^+ \frac{\varsigma_{2_{kl}}^j + \varsigma_{2_{kl}}^{j-1}}{2} = 0$$

at initial conditions

$$\varsigma_{2_{kl}}^j = \varsigma_{kl}^{j-1}, \quad \frac{\varsigma_{2_{kl}}^{j-\frac{1}{2}} + \varsigma_{2_{kl}}^{j-\frac{3}{2}}}{\tau} = q_{kl}\,. \tag{43}$$

Further we shall consider approximation

$$\frac{\varsigma_{2_{kl}}^{j-\frac{1}{2}} - 2\varsigma_{2_{kl}}^{j-1} + \varsigma_{2_{kl}}^{j-\frac{3}{2}}}{\frac{\tau^2}{4}} - \nabla_k^- C^2 \nabla_k^+ \; \varsigma_{2_{kl}}^{j-1} = 0 \tag{44}$$

Then from (44) with use (43) we receive

$$\varsigma_{2_{kl}}^{j-1/2} = \varsigma_{2_{kl}}^{j-1} + \frac{\Delta t}{2} q_{kl} + \frac{\Delta t^2}{8} \nabla_k^- C^2 \nabla_k^+ \; \varsigma_{2_{kl}}^{j-1}\,.$$

Similarly we receive algorithm of solution for the first task from (40) on the time interval $t_j \le t \le t_{j+1}$. In this case we have

$$\frac{\varsigma_{4_{kl}}^{j+1} - 2\varsigma_{4_{kl}}^{j+\frac{1}{2}} + \varsigma_{4_{kl}}^{j}}{\frac{\tau^2}{4}} - \nabla_l^- C^2 \nabla_l^+ \frac{\varsigma_{4_{kl}}^{j+1} + \varsigma_{4_{kl}}^{j}}{2} = 0 ,$$

$$\varsigma_{4_{kl}}^{j} = \varsigma_{3_{kl}}^{j} , \quad \varsigma_{4_{kl}}^{j+1/2} = \varsigma_{4_{kl}}^{j} + \frac{\Delta t}{2} q_{kl} + \frac{\Delta t^2}{8} \nabla_l^- C_{kl}^2 \nabla_l^+ \varsigma_{2_{kl}}^{j} .$$

Concerning unknown grid functions $\varsigma_{3_{kl}}^{j}$ and $\varsigma_{5_{kl}}^{j+1}$ from (39) and (40) it should be noted that also the similar equations are received, which are effectively solved by the factorization method.

To the similarly previous algorithm (the first version), in this case received scheme is absolutely steady and has the second order accuracy on time and spatial variables. At all stages of splitting the received equations are effectively solved by the factorization method.

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2.4. Some questions of considered algorithms

As a result of using of the splitting method with respect to physical processes on each time interval $t_{j-1} \le t \le t_{j+1}$ the main problem is reduced to solution of set of more simple tasks on the intervals $t_{j-1} \le t \le t_j$, $t_{j-1} \le t \le t_{j+1}$ and $t_j \le t \le t_{j+1}$.

Thus it is assumed that coefficients $\bar{u}_{k+1/2,l}$ and $\bar{v}_{k,l+1/2}$ included in the transfer-diffusion equations (at the first and third stages of splitting of the basic task), are known and constant on all interval $t_{j-1} \le t \le t_{j+1}$. Thus, on the time interval $t_{j-1} \le t \le t_{j+1}$ the values of these coefficients are determined on the previous step $t_{j-3} \le t \le t_{j-1}$ at the adaptation stage of solution of the problem (12)-(14). For definition of grid functions the following approximation are also used

$$\bar{u}_{k+1/2,l} = \bar{u}_{2_{k+1/2,l}}^{j-1/2} + \bar{u}_{5_{k+1/2,l}}^{j+1/2} \quad \text{and} \quad \bar{v}_{k,l+1/2} = \bar{v}_{2_{k,l+1/2}}^{j-1/2} + \bar{v}_{4_{k,l+1/2}}^{j+1/2}$$

Coefficients of turbulent viscosity on the interval $t_{j-1} \le t \le t_{j+1}$ at the first and third stages of splitting (transfer-diffusion stages) of the basic task are also the same, which are determined on the previous time interval by the formula [12]

$$\mu^j = \Delta x . \Delta y \sqrt{2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2}$$

where Δx and Δy are horizontal grid steps along x and y axes, respectively.

2.5. Numerical realization of the model for the easternmost part of the Black Sea

The shallow water model (SHWM) is included in the regional forecasting system for the easternmost part of the Black Sea [13-15], which is a part of the Basin-scale Black Sea nowcastong/forecstong system [16-17]. functioning of the regional forecasting system schematically is illustrated in Fig.1.

A Basin-scale model of Black Sea dynamics of Marine Hydrophysical Institute (MHI, Sevastopol) of the National Academy of Sciences of Ukraine provides RM-IG and SHWM of M. Nodia Institute of Geophysics with initial and boundary conditions on the liquid boundary. SHWM requires also wind stress components and atmosphere pressure gradients along x and y axes, which are provided from atmospheric model ALADIN. All these data with one-hour time step frequency within the 4-days period are received operatively via ftp site from MHI. Test calculations on SHWM were carried out for the same regional area, for which are calculated marine forecasts by 3-D RM-IG [13-15]. This area is bounded with the Caucasus and Turkish shorelines and the western liquid (open) boundary coincident with 39.08°E. The grid parameters are also the same for both RM-IG and SHWM: a grid has 215 x 347 points on horizons with step 1 km.

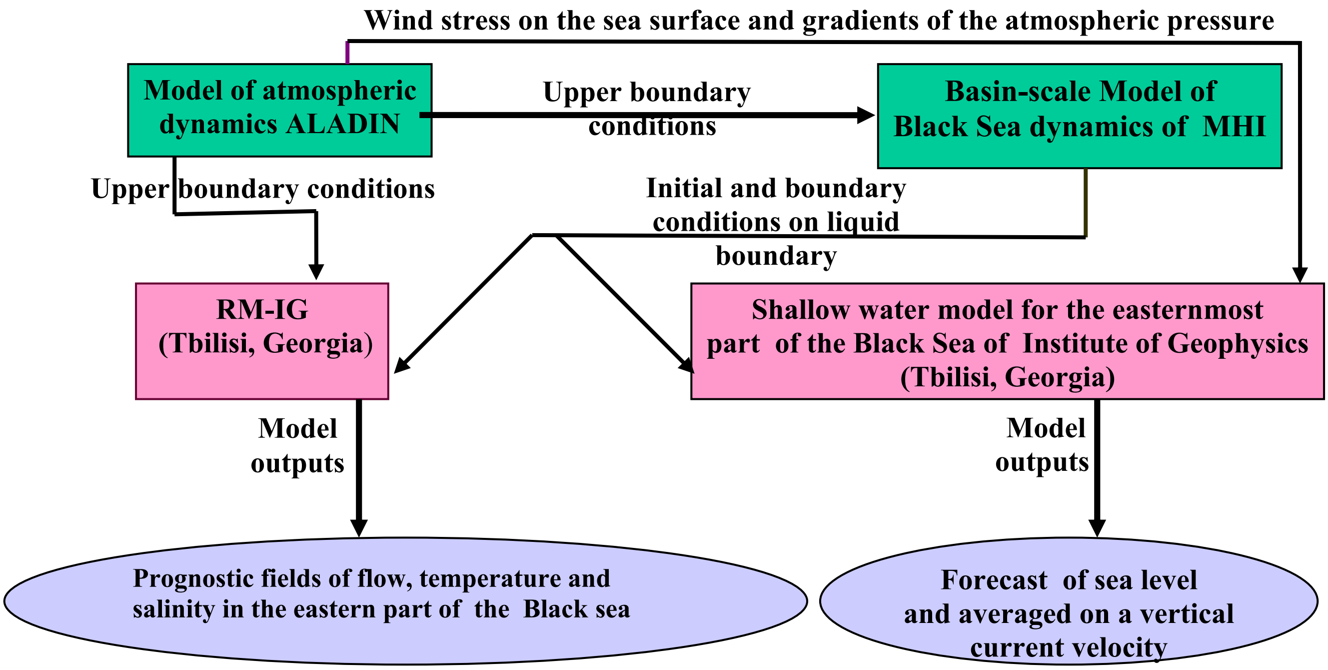
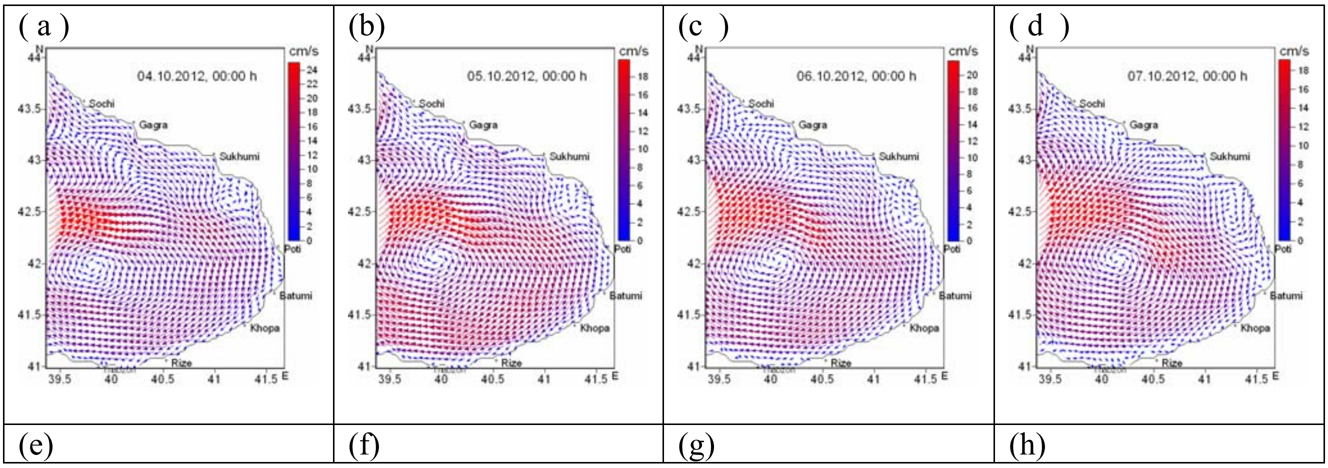


Fig.1. The scheme of functioning of the regional forecasting system.



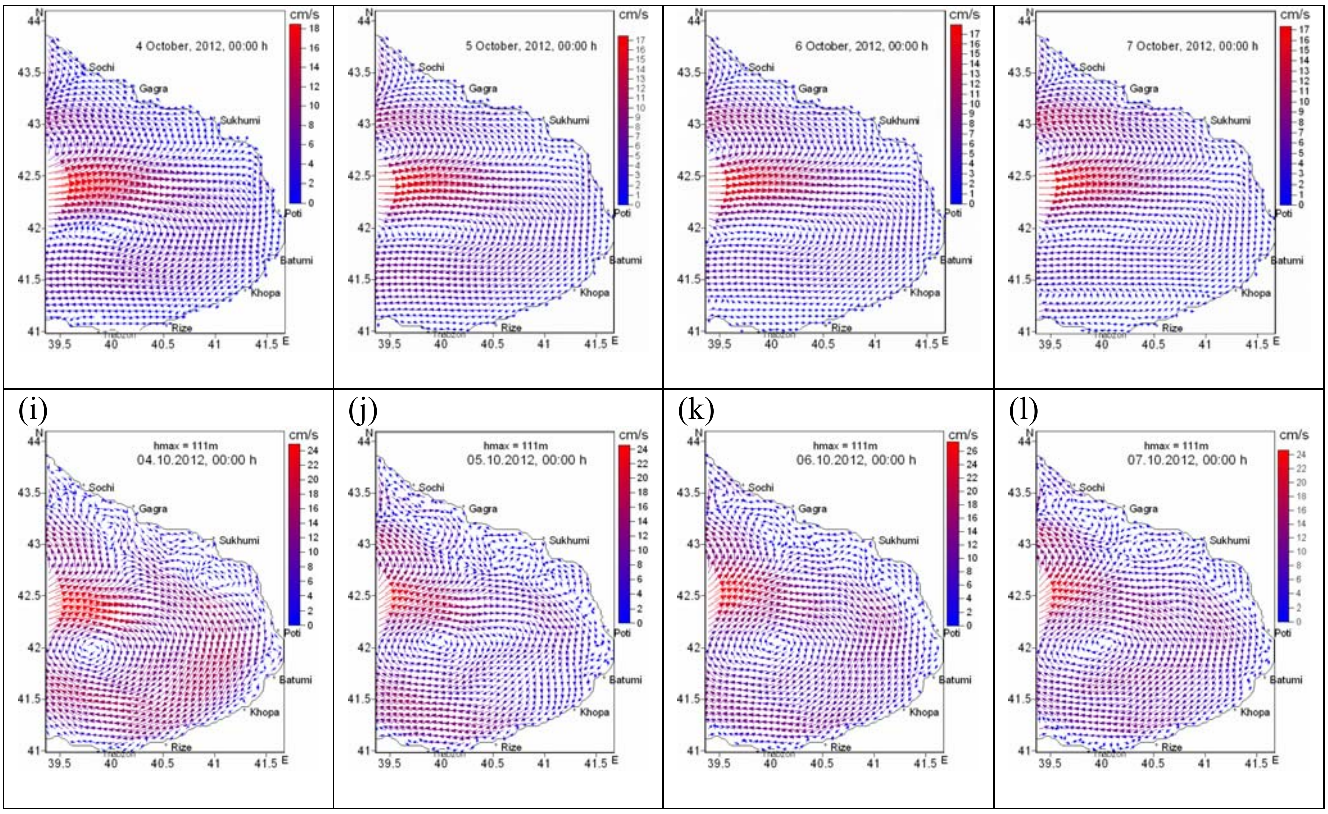
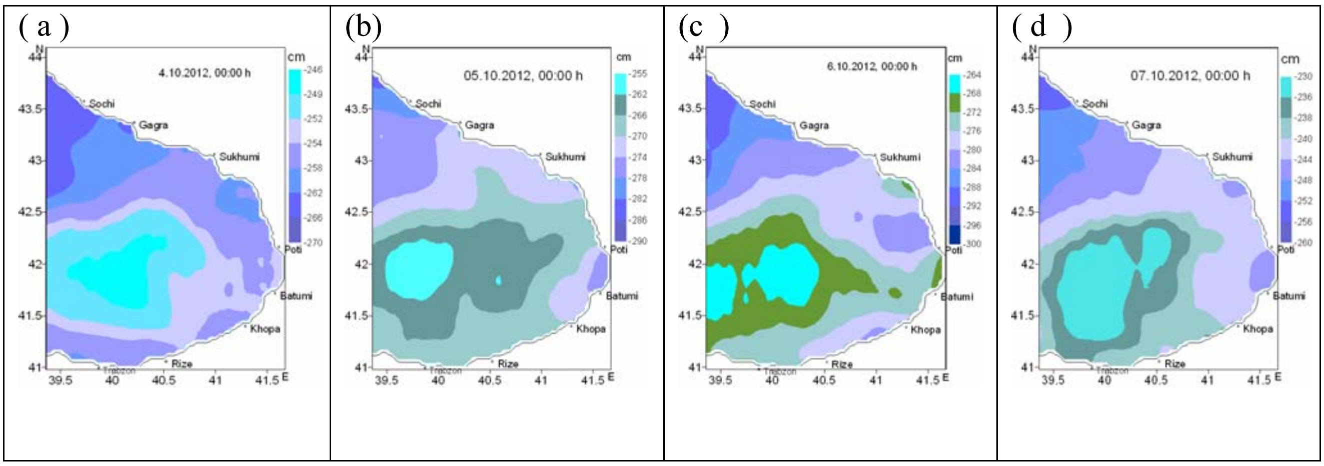


Fig.1.Simulated current fields within 4-7 October 2012. (a), (b), (c), (d) – calculated from SHWM (the first version of the algorithm), hmax = 111 m, dlt = 150 s; (e), (f), (g), (h) - calculated from SHWM (the second version of the algorithm), hmax = 111 m, dlt = 3 s; (i), (j), (k), (l) - averaged in the 111 m upper layer current fields calculated from 3-D RM-IG.

With the purpose of testing the SHWM numerical experiments were carried out by using of both versions of numerical algorithm, which are described in the previous section. The numerical experiments showed that realizations of the model with use of the second version of numerical algorithm of solution, when the equation systems at the stage of adaptation are reduced to the oscillation equations of a string, requires much less time step than in the first version. In both cases maximal depth equal to 111m was consider.



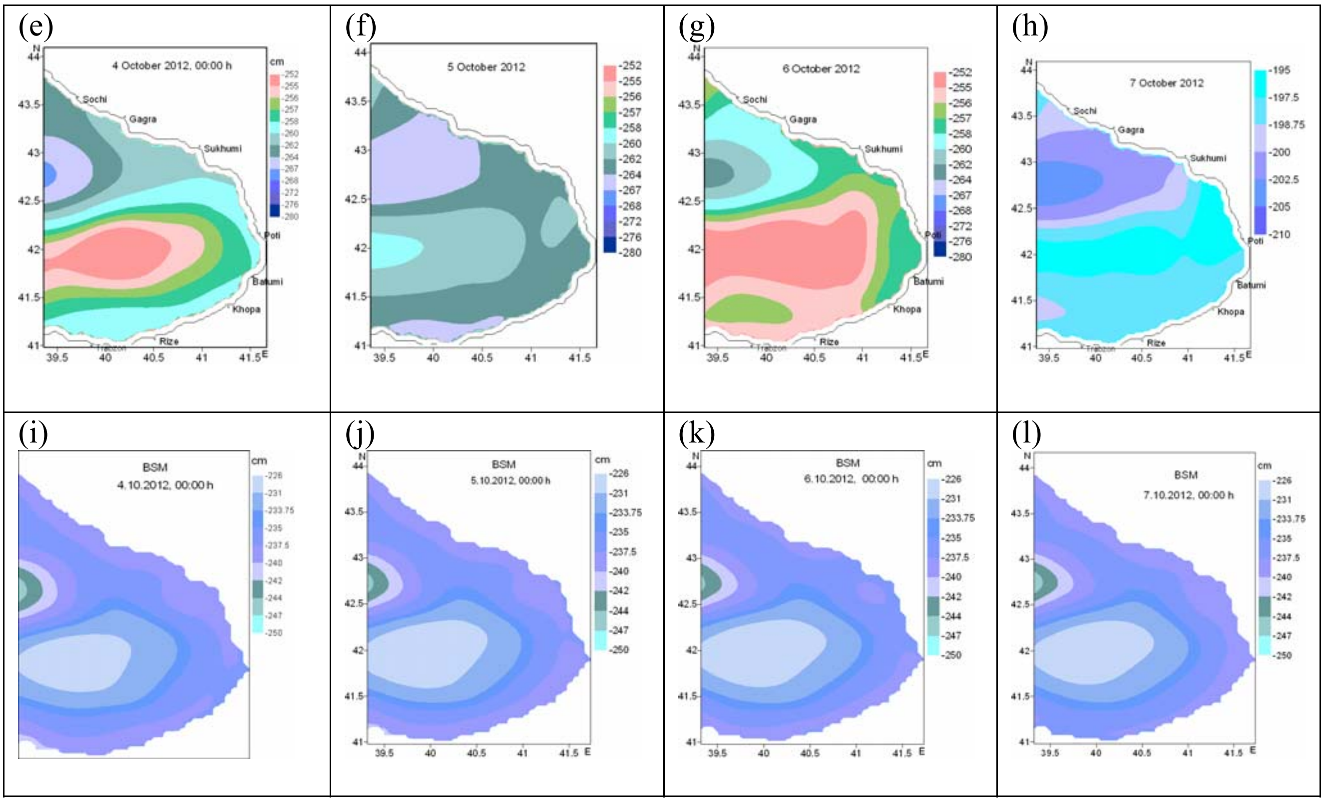


Fig.2. Simulated sea level changes (in cm) within 4-7 October 2012. (a), (b), (c), (d) – calculated from SHWM (the first version of the algorithm), hmax = 111 m, dlt = 150 s; (e), (f), (g), (h) - calculated from SHWM (the second version of the algorithm), hmax = 111 m, dlt = 3 s; (i), (j), (k), (l) - calculated from the 3-D model of Black Sea dynamics of MHI.

By SHWM the autumn circulation was simulated during 3-7 October 2012. The integration of the equations began at 00:00 h (Greenwich time), October 3 2012 and 4 days last, but we consider that the SHWM gives forecast only for three days as during the first day the coastal model runs in the prognostic mode only to have better adjustment of the fine resolution to the course initial conditions provided by the basin-scale model of MHI. At realization of the SHWM with use of the first version of the numerical algorithm for the easternmost area of the Black Sea the time step was equal to 150 s, but at realization with use of the second version the time step was 3 s.

With the purpose of illustrating in Fig.2 simulated currents are shown after 24, 48, 72, and 96 h after start of integration by the SHWM with use of the first version of the numerical algorithm (Fig.2a, b, c, d) and with use of the second version of the algorithm (Fig.2e, f, g, h), also averaged on a vertical within 111 m upper layer simulated currents by the 3-D RM-IG (Fig.2i, j, k, l). From Fig.2 it is clear that the main feature of the regional circulation for the considered time period is formation of the anticyclonic eddy (called the Batumi eddy), which covers the significant part of the considered regional area. Comparison of circulation patterns calculated from SHWM (with use of both versions of algorithm) and RM-IG shows that circulation patterns received from SHWM with use of the first version is more close to circulation patterns received from 3-D RM-IG.

In Fig.3 are illustrated simulated sea level changes after 24, 48, 72, and 96 h after start of integration using SHWM (with use of both versions of algorithm) and sea level changes received from the 3-D of MHI (Fig.2i, j, k, l). The change of the free surface level in many respects is caused by circulating features. From Fig.3 it is well visible, that in the considered area the decreasing of the sea level from the undisturbed level is observed during all time, but in the central part of the Batumi eddy the level rises. Comparison of results show that sea level patterns received from the first version of SHWM are more close to sea level patterns received from the 3-D of MHI.

Summarizing results of the carried out numerical experiments we come to opinion, that for simulation and forecast of sea coastal processes the first version of the numerical algorithm of solution of equations of the shallow water theory is more acceptable than the second version, when solution of equations on the adaptation stage of splitting is reduced to the oscillation equations. In addition, the realization of the second version requires to use very small step in time. But the consideration of this algorithm has the significant interest, as it can appear very convenient and useful to tasks concerning fluctuations and wave processes in different environments.

Finally, it should be noted that the further improvement of a regional forecasting system is connected to inclusion of modeling of coastal processes connected to change of a sea level for the shallow part of the Georgian coast with the very high spatial resolution. Thus the results of models stated in the given paper, will be used as the input data. The model takes into account mobile lateral boundary with the land, which is determined on a free surface level. It enables to predict processes of drainage and flooding.

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Об эффективных численных методах решения задачи мелкой воды. Реализация модели для восточной части Чёрного моря

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Резюме

Предложены два варианта численного метода решения системы уравнений мелкой воды, основанного на методе двуциклического расщепления, которые реализованы для восточной акватории Черного моря. В первом варианте решение ситемы уравнений, полученной в результате расщепления основной системы уравнений по физическим процессам (этап адаптации) сводится к применению метода факторизации относительно компонентов скорости течения, а во втором варианте на этапе адаптации полученная система уравнений приводится к уравнениям колебании струны относительно координатных линий, которые решаются также методом факторизации. Предложенные в данной статье методы решения не требуют применения внутренних итераций, что значительно увеличивает эффективность их практического применения.

“მცირე” წყლის ამოცანის ამოხსნის ეფექტური რიცხვითი მეთოდების შესახებ. მოდელის რეალიზაცია შავი ზღვის აღმოსავლეთ ნაწილისათვის

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რეზიუმე

განხილულია გახლეჩის ორციკლიან მეთოდზე დაფუძნებული “მცირე” წყლის თეორიის განტოლებათა სისტემის ამოხსნის რიცხვითი მეთოდის ორი ვარიანტი, რომლებიც რეალიზებულია შავი ზღვის აღმოსავლეთი აკვატორიისათვის. პირველ ვარიანტში ძირითად განტოლებათა სისტემის ფიზიკური პროცესების მიხედვით გახლეჩის შედეგად მიღებული განტოლებათა სისტემის (ადაპტაციის ეტაპი) ამოხსნა დაიყვანება ფაქტორიზაციის მეთოდის გამოყენებაზე დინების სიჩქარის კომპონენეტების მიმართ, ხოლო მეორე ვარიანტში იგივე ადაპტაციის ეტაპზე განტოლებათა სისტემა დაიყვანება სიმის რხევის განტოლებებზე საკოორდინატო ღერძების მიმართ, რომლებიც ასევე ეფექტურად ამოიხსნება ფაქტორიზაციის მეთოდის გამოყენებით. შემოთავაზებული ამოხსნის მეთოდები არ მოითხოვს შიდა იტერაციების გამოყენებას, რაც მნიშვნელოვნად ზრდის მათი პრაქტიკული გამოყენების ეფექტიანობას.