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On the convective motions in different geophysical media

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Abstract

From the unified point of view, this paper discusses some known and new results of theoretical and experimental investigations of slow mesoscale convective motions in the neutral and conductive gas and liquid mediums, and a mantle. Specific thermo-hydrodynamic regime conditions of the considered mediums were taken into account for determination of onset of convection and the vertical velocity of arisen heat thermal.

1. Introduction.

Turbulent heat- and mass-transfer processes are well known, and their research represents great interest for specialists, dealing with investigations of the geophysical and various practical problems. In that consequence, some known of other authors and some original results and discussions of the problems will be considered below [1-14]. This paper is sequential of the article [15].

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2. Double convective diffusion in **an ocean**. **Thermal and haline convection.**

2.1. Theoretical analysis of Benard's experiments was made by Rayleigh, which issued linearized thermo-hydrodynamic equations of incompressible viscous liquid in the Boussinesq approximation [5, 7]

$$\frac{\partial v_i}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x_i} + \nu\Delta v_i + g\alpha T\delta_{i3}, \quad \frac{\partial\theta}{\partial t} = -Aw + \nu_\theta\Delta\theta, \quad \frac{\partial v_i}{\partial x_i} = 0, \qquad (1)$$

where $A = (T_2 - T_1)/h$, $T = T_0(z) + \theta$, $T_0 = T_1 + Az$.

For solution of the Benard problem Rayleigh introduces the idea of free layer without tension forces at its boundaries:

$$\theta = w = \frac{\partial^2 w}{\partial z^2} = 0 \quad \text{at} \quad z = 0 \text{ and } h. \qquad (2)$$

Seeking the solution in the form normal modes Rayleigh found exact solution and obtained the critical values of the introduced parameter $Ra = g\alpha\Delta Th^3/\nu\nu_\theta$, $Ra_c = 27\pi^4/4 = 657.511$ (of the onset of thermal instability), and wave parameters ($|k|, n$) of the most fast growing perturbation.

$$(v_i, p', \theta) = e^{-\lambda t + i(k_1x_1 + k_2x_2)}(\hat{v}'(z), \hat{p}'(z), \hat{\theta}(z)), \qquad (3)$$

$$\hat{w}(z) = w_0 \sin n\pi z \,. \qquad (4)$$

Breaking stability depends on the form and dimensions of convective motions, perturbations scale, conditions at the boundaries of convective layer etc. The Rayleigh criterion is a criterion of onset and evolution of the cellular convection. At the critical value of the Rayleigh number, periodical relative to the spatial values stationary convective motions are arisen.

2.2. The temperature and salinity of sea water has non-uniform distribution. Most essential peculiarity of the sea water is influence of heat and salt on the density and characteristic property of heat and salt. Their opposite action upon the density of the sea water is reflected in expression $\Delta\rho/\rho_0 = \beta\Delta S - \alpha\Delta T$. Diffusion of a heat and salt in the sea water is determined by the thermal diffusivity ($\nu_T \approx 1.5\cdot 10^{-3}\, cm^2 s^{-1}$) and the diffusion constant ($\nu_S \approx 1.3\cdot 10^{-5}\, cm^2 s^{-1}$), $\nu_T/\nu_S \approx 115$. Brilliant mental experiment of Stommel-Arons-Blanchard (1956) [6], short report "An oceanographical curiosity: the perpetual salt fountain" become basic work initiated and stimulated study of convection in the presence of double diffusion process. The authors also suggested a scheme of operation.

In the sea, where the double convective diffusion takes place, the Archimedes force of total buoyancy has following form [7]:

$$g\frac{\Delta\rho}{\rho_0} = g(\alpha\Delta T - \beta\Delta S), \qquad R = \frac{\alpha|\Delta T|}{\beta|\Delta S|}, \qquad (5)$$

where $\Delta\rho = \rho - \rho_0$, $\Delta T = T - T_0$, $\Delta S = S - S_0$, here ρ_0, T_0, S_0 are means of the density, temperature and salinity at the layer lower boundary, $\Delta\rho$, ΔT, ΔS are increments of the density, temperature and salinity at the layer upper boundary; α and β are coefficients of volumetric expansion and its salt analogue, respectively; R is so called relation of buoyancy, which characterizes influence of two diffusive components upon the density change.

It is said that the layer is stably stratified if $\Delta\rho < 0$; when $\Delta\rho = 0$, stratification of layer is neutral; when $\Delta S = S = 0$ – thermal convection (heating from below), then $\Delta T < 0$ and $\Delta\rho > 0$ stratification of layer is unstable; when $\Delta T = 0$ – isothermal salt convection (salinization from above), then $\Delta S > 0$ and $\Delta\rho > 0$ – stratification of layer is unstable, too.

In the non-trivial double-convection case when $\Delta T \neq 0$, $\Delta S \neq 0$ it is evident that four cases of the temperature and salinity drops may be considered (salinization and heating of the water layer from above or from below):

(1) The salinization and heating from above: $\Delta S > 0$, $\Delta T > 0$, $\Delta\rho/\rho_0 = -\beta\Delta S(R-1)$;

(1a) $R > 1$, $\Delta\rho < 0$, stable stratification, (1b) $R < 1$, $\Delta\rho > 0$, unstable stratification.

(2) The salinization and heating from below: $\Delta S < 0$, $\Delta T < 0$, $\Delta\rho/\rho_0 = \beta\Delta S(1-R)$;

(2a) $R < 1$, $\Delta\rho < 0$, stable stratification, (2b) $R > 1$, $\Delta\rho > 0$, unstable stratification;

(3) The salinization from above and heating from below. $\Delta S > 0$, $\Delta T < 0$, $\Delta\rho/\rho_0 = \beta\Delta S(R+1)$
– stratification always is unstable;

(4) The salinization from below and heating from above. $\Delta S < 0$ $\Delta T > 0$, $\Delta\rho/\rho_0 =$
$= \beta\Delta S(R+1) < 0$ – stratification always is stable.

Expressions for convective vertical velocity and effective Reynolds number have following form [7]

$$w = \frac{2a_T k_\perp}{\tau\delta}\left[\left(1+\delta^2\right)\left(\sqrt{r}-1\right)\right]^{1/2}, \quad \text{or} \quad w = \frac{2a_s k_\perp}{\delta}\left[\left(1+\delta^2\right)\left(\sqrt{r}-1\right)\right]^{1/2}, \qquad (6)$$

where $\delta = l^*/H$, $k_\perp = \pi/H$, $\sigma = \Pr = \nu/a_T$, $\tau = a_T/\nu_s$, $r = \frac{\tau}{R} = \frac{a_T}{a_s}\frac{\beta\gamma_s}{\alpha\gamma_T}, \gamma_T = \frac{dT}{dz}, \gamma_s = \frac{ds}{dz}$,

where

$\Pr \approx 6.93$, $\tau \approx 115$; for $T_0 = 20^0 C$, $s_0 = 40\%$, $\beta = 0.72$, $\alpha = 2.1\cdot 10^{-4}(^0 C)$. (7)

In the laboratory experiments modeling a double-convection in the systems heat-salt and sugar-salt (similar systems NaCl-KCl) no diffusing components loss takes place.

According to measurements in the upper layer of the ocean the vertical velocity of convectional motion changes from 0.04 $cm\cdot s^{-1}$ at the surface (h = 0 m.) of the ocean to 0.91 $cm\cdot s^{-1}$ at the depth h = 600 m.

$$w = 0.04 \div 0.91 \ cm\cdot s^{-1}, \quad h = 0 \div 600 \ m. \qquad (8)$$

2.2.2. In the presence of vertical velocity shear of the main fluid flow dU/dz in the layer of the water with heterogeneous density with a thickness equals to h^* respective Rayleigh number Ra^* is, [8],

$$Ra^* = \frac{h^{*3}}{h^4}\frac{1}{\nu}\left|\frac{dU}{dz}\right| Ra; \qquad Ra^* \le Ra. \qquad (9)$$

Here h is the initial thickness of the free convection layer. Positive values of the Rayleigh number $Ra^* > 0$ characterizes instability of the layer h^*.

3.1. Two-phase flows [16]. Equation of a turbulent energy balance in the shearing motion containing solid particles of sparse distribution takes up the form [15]

$$\rho < u'w' > \frac{\partial u}{\partial z} + \rho\varepsilon_t + < \rho'w' > g = 0; \qquad (10)$$

it is convenient to rewrite the aforecited equation in a following shape

$$< u'w' > \frac{\partial u}{\partial z}(1 - Ko) + \varepsilon_t = 0, \qquad (11)$$

where the non-dimensional **K**olmogorov parameter, *Ko*,

$$Ko = -\frac{\overline{\rho'w'}\,g}{\rho\overline{u'w'}\,u_z} = -\frac{\sigma g\,\overline{s'w'}}{\overline{u'w'}\,u_z}, \qquad 0 < Ko < 1, \qquad (12)$$

u', w', ρ', and s' are pulsations of horizontal and vertical components of velocity, density of mixture, and mean volume concentration, s, of the particles, respectively; $\sigma = (\rho_p - \rho)/\rho$, ρ_p is the density of particles, and $u_z = \partial u/\partial z$. The Kolmogorov number shows a turbulent energy consumption of the flow to the weighing of particles. When the Kolmogorov number *Ko* ~1, the particles influence on the dynamics of the flow is great, i.e., the Kolmogorov parameter *Ko* becomes an additional parameter determining influence of the stratification. **Thus, the Kolmogorov parameter, *Ko*, is similar to Richardson's one in the theory of temperature stratification.**

3.2. The energetic layer of an ocean.

(a) (b)

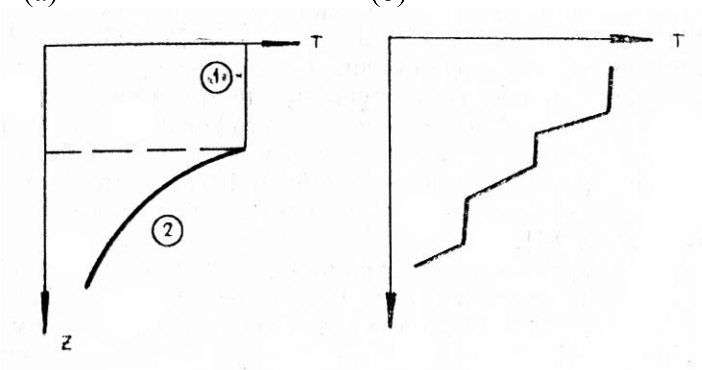


Fig. 1. Temperature distribution in the upper layer of the ocean:
(a) schematic profile of temperature: (1) is the upper uniform layer,
(2) is the upper thermocline [16]; (b) precise measuring profile [17].

Upper energetic layer of an ocean is uniform (in it the temperature and salinity. and, therefore, the density are constant) is connected with the turbulent mixing (see Fig.1). The mixing is realized by the wind shear and convection: descending heavy particles to swim with the current from the upper layer, cooled and salted, as a result of evaporation from the surface and also of breaking of the surface waves. The depth of this layer depends on the season: it increases in winter and decreases in spring. The upper uniform layer is supported by the region with sharply changed temperature (Fig. 1a) – upper thermocline to the depth about 200-250 m. Here a seasonal temperature changing does not become apparent. An analysis of temperature distributions in the strong and stable stratified upper thermocline shows that in the upper thermocline the turbulent diffusivity coefficient is of order $æ_t \sim (10^{-1} \div 1)\, cm^2/s$, intermediate value between the upper turbulent uniform layer's value equal to $æ_t \sim 10^3\, cm^2/s$, and the value of the molecular thermal diffusivity, $æ \sim 10^{-3} cm^2/s$. More precise measurements [17] obtain, that instantaneous temperature stratification has stepped character: the range of constant temperature changed by region with great gradients (Fig. 1b). That results from turbulence in the turbulent flow with steady stratification is spread in the form of pots and connected with internal waves [18].

3.3. Laboratory experiments [19]. Free convective motions inside of the heterogeneity liquids are one of main processes giving rise to generation of fine structure of ocean, atmosphere, and, seemingly, only mechanism of stratification of Antarctida' Lake Vanda type closed basins. Laboratory experiments (provided under lateral heating) showed that general property of the spatial structure (SS) of convective motions inside stratified liquid (SL) is generation periodic vertical circular layers. These layers separated by thin plates of still liquid having great gradients of temperature and salinity, shear of velocity. The convective processes inside of SL are convenient model for study of periodical SS dynamics inside of heterogeneity media. In this case, the temperature is "fast" variable and salinity is "slow" variable, spatial dispersion caused by difference of their kinetic coefficients – the thermal diffusivity $\chi = 1.43 \cdot 10^{-3} cm^2/s$ and diffusion of a salt $k_s = 1.41 \cdot 10^{-5} cm^2/s$ [19].

4. Golitsyn's approximate theory of the roll convection [11].

4.1. The Ū-Ra-Nu relation.An interest to this problem was arisen under attempts to estimate the motion velocities in the Earth upper mantle, which cause displacements of the lithospheric plates of the Earth's crust. From the formula for the energy dissipation (under equal derivatives of the velocity components) and the character scale equal to the thickness of the layer d (roll convection) the mean values of the velocity components are [11]:

$$\bar{u} \approx \bar{w} \approx \frac{1}{a}\left(\frac{\varepsilon}{\nu}\right)^{1/2} d = \frac{d}{a}\left(\frac{agf}{\mu c_p}\frac{Nu-1}{Nu}\right)^{1/2}, \qquad (13)$$

or

$$\bar{u} \approx \bar{w} \approx \frac{\kappa}{ad}\left(Ra(Nu-1)\right)^{1/2}, \qquad Nu >> 1. \qquad (14)$$

i.e. for sufficiently developed convection, when the Raynolds number $\mathrm{Re} \geq 1500$, the Rayleigh number $Ra \sim 10^7$ and between them is following relation:

$$\mathrm{Re} = 0.5 a_1 P^{-2/3} Ra^{4/9}, \qquad (15)$$

where a_1 is experimental constant, κ is the coefficient of thermal diffusivity.

The author using the McKenzie et al. (1974) [11] recommendations for values of the material parameters: $\alpha = 2\cdot 10^{-5} K^{-1}$, $c_p = 1.2\cdot 10^3 J/(kg\cdot K)$, $\rho = 3.6 t./m^3$, $\nu = 2\cdot 10^{17} m^2/s$, obtains for the geothermal flux of heat value $f = 6\cdot 10^{-2} Wt/m^2$, for thickness of the upper mantle value $d = 7\cdot 10^5 m = 700 km$, and for mean velocity (14) following value

$$\bar{u} \approx 1 cm/year. \qquad (16)$$

According to Elsasser et al. (1979) ([11]) all mantle with thickness $d \approx 3000 km$ takes part in convection process. Then

$$\bar{u} \approx 5 cm/year, \qquad (17)$$

what is near to real situation

$$\bar{u} \approx 10 cm/year. \qquad (18)$$

Another available conclusion for study fluid motions in the mental is that under small Raynolds' numbers the self-similarity of convection follows from these conceptions [12, 13]. *There is possibility of laboratory modeling of such motions under small Reynolds number* [11]. In detail, results of the laboratory investigations of this problem are discussed in the large paper [14].

4.2. The Nu-Ra relation.

Thermal conductivity equation in a fluid without internal sources,

$$\frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i} = \kappa \frac{\partial^2 T}{\partial x_i^2}, \qquad (19)$$

is a homogeneous equation relative to the choice of the temperature scale. Introduce the scales of the length, d, velocity, U, and time, d/U. Then before the Laplacian in r.h.s. is appeared a factor, Pe^{-1} (where Pe is the Peklet number),

$$Pe = Ud/\kappa. \qquad (20)$$

It is evident that when $Pe >> 1$ the thermal boundary layer of the thickness

$$\delta \approx d\, Pe^{-1/2}, \; (\frac{d}{\delta} \approx Pe^{1/2}), \qquad (Nu \sim \frac{d}{\delta}), \qquad (21)$$

is generated, but in the main part of liquid its temperature must little change.

A lot of laboratory and numerical experiments [20] confirm this picture and show that at the developed convection the temperature changes about $\Delta T/2$ near the boundaries and in main volume the liquid is isothermal, practically.

From the heat flux continuity through the liquid it is evident that $f/\rho c_p \approx \kappa \Delta T / 2\delta$. Using (21) and $fd = \rho c_p \kappa \Delta T + HG$, $H = c_p / \alpha g$, the author obtained the relation [11]:

$$Nu \approx Pe^{1/2}/2. \qquad (22)$$

Choose the scale of velocity in form of (14), then instead of (22) we have

$$Nu \approx [Ra(Nu-1)]^{1/4}/2a^{1/2}. \qquad (23)$$

At small supercritical values of the Rayleigh number $Ra(Nu-1) \sim Ra - Ra_c$, i.e.

$$Nu \sim (Ra - Ra_c)^{1/4}. \qquad (24)$$

For Nu>>1, from (23) we have that

$$Nu \sim 2^{-4/3} a^{-2/3} Ra^{1/3}. \qquad (25)$$

These heat-transfer principles are well-known experimentally and the last one also has theoretical substantiations [20, 21]. *These formulas (22)-(25) are free from several assumptions of other authors*. Coefficient $2^{-4/3}a^{-2/3}$ = 0.1 at a = 9 and 0.08 at a = 12.

According to [22], in case of plane surface for $Ra > 10^9$ at united laminar and turbulent convection $Nu = 0.13 Ra^{1/3}$, for Ra $10^4 \div 10^9$ at a natural laminar convection the dependence is lower $Nu = 0.59 Ra^{1/4}$, but according to [14] $Nu \approx 0.1 Ra_\gamma^{1/3}$. Last investigations reviewed in [14] confirm Nu - Ra relation obtained by [11].

4.3. The K-Qτ relation..

For non-dimensional kinetic energy of convection, K, it is obtained the formula

$$K = \frac{1}{2}(\overline{u}^2 + \overline{w}^2) \approx \frac{Ra(Nu-1)}{a^2}, \qquad (26)$$

in dimensional form

$$K \approx \frac{\gamma f}{a^2}\frac{d^2}{\nu} = \frac{G}{a^2}\tau_{rel}, \qquad (27)$$

where $G = \gamma f$ is the velocity of the kinetic energy generation from potential one (mechanical power), $\gamma = dT/dz$, f is the heat flux through the liquid without internal sources of heat, $\tau_{rel} = d^2/\nu$ is the viscous relaxation time.

For density convection in (14) instead of $G = \gamma f$ one can substitute $G = \gamma_g mgd$, where γ_g is the velocity of the kinetic energy generation, m is the flux of density through the layer, mgd is the mechanical power introducing into the flux under a stationary density convection.

In the inertial interval (homogeneous isotropic interval) for kinetic energy of the volume with a mass $M = \rho d^3$ relative to similar neighbour volumes the kinetic energy of convection

$$K \approx M(\varepsilon d)^{2/3} = Q_m d(\varepsilon d)^{-1/3}, \qquad (28)$$

where $Q_m = \rho \varepsilon d^3$ is the incoming total power of energy from the external scale of turbulence and dissipating into the viscous interval. Because of $(\varepsilon d)^{1/3} \approx U$, the mean square different of velocities in two points of the area divided by the distance equals to d , than $d(\varepsilon d)^{-1/3} = \tau_U$ is suggested as the character time life of the vortex of the scale d .

$$K \approx Q_m \tau_U . \qquad (29)$$

The kinetic energy of circulation on the slowly revolving planet may be written in similar form

$$K \approx Q\tau_e , \qquad (30)$$

where $Q = 4\pi a^2 q$ is a total energy surge to the planet of radius a , q is a mean solar energy surge to the area element of a planet surface, $\tau_e = a / c_e$ is a character time of propagation of disturbance in global scale, c_e is the isothermal speed of sound at the equilibrium temperature $T_e = (q/\sigma)^{1/4}$, and σ is the Stephan-Boltzmann constant.

Last formulas show that the total kinetic energy of large quantity of forced flows is determined by the product of the energy input into the liquid and the character time of relaxation. It is of importance that in all considered cases above mentioned time is the least among all times which may construct from parameters of the problem (having at one's disposal). Taking into account the hypothesis of self-similarity, that time is generally single one. Golitsyn introduces the rule of the fastest reaction which he names as "*principle of the fastest reaction*": *the kinetic energy of constant forced flow is of the order of power input multiplied by the minimal relaxation time character for the system.* This rule allows without recourse to the similarity theory to write the expression for total kinetic energy of the system.

5. The mantle plumes.

5.1. A simple model for planetary mantle convection is the Bénard convection in a fluid with a temperature-dependent viscosity. In the Bénard problem, dissipative processes play an essential role. Bénard was particularly interested in the role of viscosity. He found that when the temperature of the lower surface was gradually increased, at a certain instant, the layer became reticulated and revealed its dissection into cells [9]. This problem is one of the actual problems of the Geophysics and Physics of the Earth.

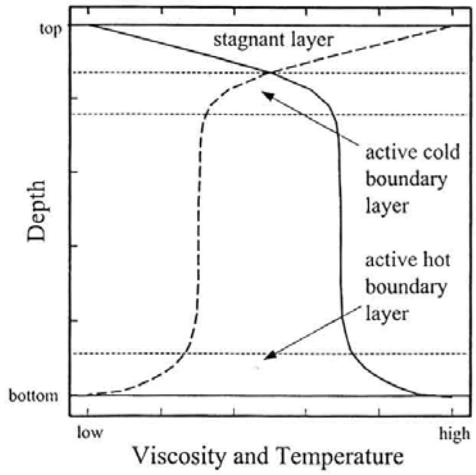


Fig. 3. A schematic illustration of the horizontally averaged variation of temperature (solid line) with depth during an experiment. Also shown are the active thermal boundary layers (thin dashed lines) at the top and bottom of the fluid layer. The high viscosity of the coldest region makes the upper part of the cold thermal boundary layer stagnant. Resultant weak cooling keeps the actively convecting region nearly isothermal and, in turn, the viscosity ratio across the hot thermal boundary layer small [14].

5.2. According to [23] the main unknown parameters are viscosity values within mantle layers whose number and thickness are prescribed in the models developed during the past two decades. These parameters are estimated by comparing observations and predictions of relative sea level change at various sites over the past 18,000 years. Secular changes of length of day and the Earth's gravitational oblateness also contain information on the depth-varying mantle viscosity. The upper mantle viscosity is fixed here at $5 \cdot 10^{20}$ Pa s. Comparison of theoretical oblateness- viscosity curve and observational one with each other leads to a lower mantle viscosity around $2 \cdot 10^{21}$ Pa s. (The author suggests the range $1-5 \cdot 10^{21}$ Pa s that fits the geological observations). This evident is conformed to the diagram in Fig. 3 [14].

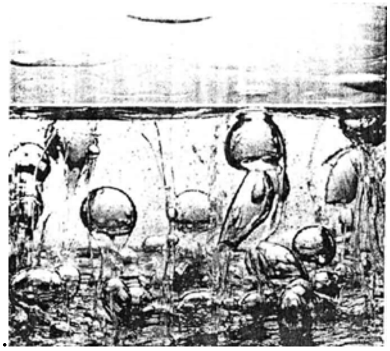


Fig. 3.

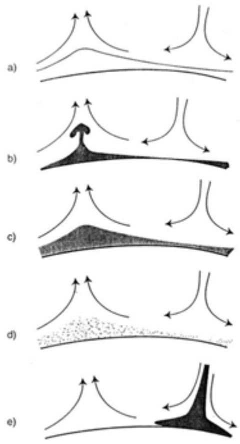
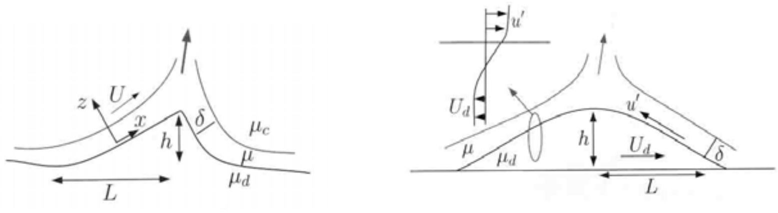


Fig. 4.

Fig. 3 (left). A laboratory experiment with compositional convection in which low-viscosity water is injected through a permeable plate into high-viscosity glucose syrop. In a way that is dynamically similar to thermal convection, water collects in a gravitationally unstable compositional boundary layer at the base of the syrup and then drains intermittently as plumes with large heads and narrow underlying conduits. Despite the presence of robust low-viscosity conduits, complicated interactions among rising plumes prevent their becoming long-lived stable features [14].

Fig. 4 (right). Schematic illustration of several models for D″. Within the context of plate tectonics, D″ has been explained variously as (a) a phase change, (b) a thermal boundary layer, (c) a compositional boundary layer, (d) ponded chemical dregs from subducted lithosphere, and (e) a slab graveyard [14].



(a) (b)

Fig. 5. (a) Schematic cross section of the deformed dense layer defining variables and the geometry of the problem – dense layer topography and long-lived plumes. This scheme was constructed on the basis of laboratory experiments which showed as the dense layer is deformed by flow into a nascent plume instability showing the different regions of the flow. (b) Schematic cross section of the deformed dense layer defining variables and the geometry of the problem – height of topography. In order for topography to be stable $U_d \sim u'$ [14].

5.3**.** According to [26] lubrication theory analysis the perturbed velocity of fluids [14]

$$u(z) = U + u'(z), \tag{31}$$

where U is the velocity at the boundary between the interior and thermal boundary layer fluid, $u'(z)$ describes the variations in velocity within the boundary layer. The *x*-component of the momentum equation is

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u'}{\partial z^2}, \qquad \text{and} \qquad \frac{\partial p}{\partial x} \approx \frac{\Delta \rho g h}{L}, \tag{32}$$

where *p* is dynamic pressure, we have

$$u' \sim \frac{\Delta \rho g h \delta^2}{\mu L}, \tag{33}$$

Continuity of viscous stresses at the interface between the cold interior fluid and the thermal boundary layer demands that $\mu_c U / L \sim \mu u' / \delta$ and thus that

$$U \sim \frac{\Delta \rho g h \delta}{\mu_c}. \tag{34}$$

The criterion for stable plumes is that the velocity *U* must be greater than the speed, at which a thermal can rise through the mantle,

$$U_{th} \sim \frac{\Delta \rho g \delta^2}{\mu_c}, \tag{35}$$

where $\Delta\rho$ is the difference between the density of thermal boundary layer fluid and interior fluid, g is gravity μ_c is the viscosity at the cold boundary, δ is the thermal boundary layer's thickness, h is the dense layer's height. This condition leads to the requirement that $h/\delta > const$ (in the experiments $h/\delta \approx 0.6$).

Long-lived plumes are located on top of topographic peaks on the dense layer. In order for a plume conduit to become fixed on top of such a feature it is clear that thermal boundary layer fluid must flow along the interface with the dense layer faster than it can rise vertically into the interior as a new thermal. Said differently, the timescale for thermal boundary layer fluid to flow laterally from the center of an embayment to a peak must be less than the timescale for a new convective instability to grow.

The scheme Fig. 5a was constructed on the basis of laboratory experiments which showed as the dense layer is deformed by flow into a nascent plume instability showing the different regions of the flow. Knowing the height of the topography on the dense layer is critical for determining the stability of plumes. One dynamical requirement for stable topography is that the lateral flow of boundary layer fluid must be balanced by the opposing flow of dense layer material (Fig. 5b). The condition implies that $U_d \sim u'$, where $U_d \sim \Delta\rho g h \delta^2 / L\mu$ and thus that

$$\frac{h}{\delta} \sim \left(\frac{\Delta\rho}{\Delta\rho_c}\right)^{1/2} \sim \left(\frac{1}{B}\right)^{1/2}. \qquad (36)$$

6. Some remarks.

For comparison with the above mentioned picture of air bubbles generation (Fig. 3), below it is given the similar picture of air babbles, generated in super-cooled water drop after its freezing. Water drops of about 2-3 mm in diameter were frozen on the ice in the original micro-cold-store engineered in Geophysical Institute of Georgian Acad. Sci. [28].

Fig. 6. Internal bubble structure of frozen supercooled water drop of about 2-3 mm in size [28].

were used. After freezing maximal air babbles' diameter was about 0.3 mm. The speed of spreading of the crystallization front $G \approx 3.48 \cdot 10^{-3}\, cm \cdot s^{-1}$; temperature drop of air in the cold store $\Delta T = -10^0$, thermal conductivity of air $\lambda_a = 5.63 \cdot 10^{-5}\, cal(cm \cdot sec \cdot K)^{-1}$, solubility of air in water in mole fractions is $D \sim 1.3 \div 1.8 \cdot 10^{-5}$; diffusion of a heat in the water $\nu_T \approx 1.5 \cdot 10^{-3}\, cm^2 s^{-1}$. The sizes of the air bubbles and thickness of initial ice layers in the freezing drop correlate with D/G. The thickness of first clear layer of ice is about 0.1-0.2 mm, and diameters of air bubbles ~ 0.12-0.16 mm.

6.1. It necessary to note, that criterion $B = \Delta\rho / \rho\alpha\Delta T$ in [14] may be obtained as ratio of criteria Archimedes, $Ar = (gl^3 / \nu^2)\Delta\rho / \rho$, and Grashof criteria, $Gr = (gl^3 / \nu^2)\alpha\Delta T$:

$$Ar : Gr = B = \Delta\rho / \rho\alpha\Delta T \,. \qquad (37)$$

When the Reynolds number $\mathrm{Re} = Ul / \nu$ equals to the Archimedes number $Ar = (gl^3 / \nu^2)\Delta\rho / \rho$, then for an ascending motion velocity of the warm mass of liquid we have the following formula:

$$U = \frac{\Delta\rho g l^2}{\rho\nu}; \qquad (38)$$

Using the relation $U / L \approx u' / h$ or $U \approx u'L / h$, i.e. we have that $u'L / h \approx \Delta\rho g l^2 / \rho\nu$ or $u' \approx \Delta\rho g h l^2 / \rho\nu L$. Having suggest $l = \delta$, one can obtain the expression (35) for the velocity variation in the boundary layer according to the modeling experiments [14]:

$$U_{th} \sim \frac{\Delta\rho g \delta^2}{\mu_c}.$$

6.2. Let compare Golitsyn's formula (22) with Jellinek-Manga's one (36):

$$Nu \approx \frac{d}{\delta} \sim Pe^{1/2} \quad \text{and} \quad \frac{h}{\delta} \sim B^{-1/2},$$

we obtain dependence between the Peklet number, and the form-factor B

$$Pe \sim B^{-1}$$

here $d = h$ $Pe = Ud / \kappa$, and $B = \Delta\rho_c / \rho\alpha\Delta T_i$ is the stabilizing buoyancy effect of the dense layer.

6.3. The theory of thermal instability in fluid spheres and in spherical shells has bearings on a number of geophysical questions [9]. Though applications of the theory are not universally subscribed, it cannot be doubted that convective motions in the fluid core are relevant to all theoriesconcerned with the origin of the earth's magnetic field and its secular variations.
But the theory of thermal instability has not been worked out with sufficient generality for these purposes. Even the effect of rotation has been examined only in a very preliminary way; and the onset of instability as overstability – which should be expected to be the rule rather than the exception with liquid metals requires investigation. And in addition to rotation, the effect of a magnetic field has also to be considered. The case of a uniform magnetic field presents no formal difficulty; but this is hardly appropriate for the problems in view. Without further knowledge, the choice of an initial field is so wide that the selection becomes almost arbitrary. It is, indeed, likely that the theory of the convective motions in the earth's core cannot be dissociated from the theory of the origin of the earth's magnetic field.

7. Conclusion.

In the above considered cases of convective motions we practically deal with the Bénard problem: (a) for a single fluid when the instability has a simple mechanical interpretation and (b) for a mixture in which the motion gets complicated by the diffusion processes. In the linear stability theory, it is generally assumed that the most general perturbation can be represented as a complete set of normal modes. This approach of the problem, as is well known, was carefully developed by Chandrasekhar [9], and analyzed later, for example, in the monographs [10, 27]), especially by Joseph [10].

Being first step in analysis of the convective motions the linear theory cannot answer a number of essential questions. First of all, that is a question about stabilization of the rapidly increasing perturbations, secondly, a question about the structure of convective cells and their stability. Only by means of non-linear theory it is possible to answer these questions.

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О конвективных движениях в различных геофизических средах

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С единой точки зрения рассматриваются результаты теоретических и экспериментальных исследований медленных мезамасштабных конвективных движений в атмосфере, океане и мантии. Учтена специфика режимов рассматриваемых сред при определении условий возникновения конвекции и нахождении аналитических формул для вертикальной скорости восходящего термика.

კონვექციური მოძრაობების შესახებ სხვადასხვა გეოფიზიკურ გარემოში

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რეზიუმე

განხილულია ოკეანესა და მანტიაში ნელი კონვექციური მეზომასშტაბური მოძრაობების თეორიული და ექსპერიმენტული შესწავლის შედეგები. კონვექციის წარმოშობის პირობებისა და აღმავალი თერმიკის ვერტიკალური სიჩქარის ანალიზური ფორმულების განსაზღვრისას გათვალისწინებულია განხილულ გარემოთა რეჟიმების სპეციფიკა.