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Generation, intensification and self-organization of internal-gravity wave structures in the Earth's ionosphere with directional wind shear

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Abstract

The linear mechanism of generation, intensification and further nonlinear dynamics of internal gravity waves (IGW) in stably stratified dissipative ionosphere with non-uniform zonal wind (shear flow) is studied. In case of the shear flows the operators of linear problem are non-selfadjoint, and the corresponding Eigen functions - nonorthogonal. Thus, canonical - modal approach is of less use studying such motions. Non-modal mathematical analysis becomes more adequate for such problems. On the basis of non-modal approach, the equations of dynamics and the energy transfer of IGW disturbances in the ionosphere with a shear flow is obtained. Necessary conditions of instability of the considered shear flows are obtained. The increment of shear instability of IGW is defined. Exact analytical solutions of the linear as well as the nonlinear dynamic equations of the problem are built. It is revealed that the transient amplification of IGW disturbances due time does not flow exponentially, but in algebraic - power law manner. The frequency and wave-number of the generated IGW modes are functions of time. Thus in the ionosphere with the shear flow, a wide range of wave disturbances are produced by the linear effects, when the nonlinear and turbulent ones are absent. The effectiveness of the linear amplification mechanism of IGW at interaction with non-uniform zonal wind is analyzed. It is shown that at initial linear stage of evolution IGW effectively temporarily draws energy from the shear flow significantly increasing (by order of magnitude) own amplitude and energy. With amplitude growth the nonlinear mechanism of self-localization turns on and the process ends with self-organization of nonlinear solitary, strongly localized IGW vortex structures. Therefore, a new degree of freedom of the system and accordingly, the path of evolution of disturbances appear in a medium with shear flow. Depending on the type of shear flow velocity profile the nonlinear IGW structures can be the pure monopole vortices, the transverse vortex chain or the longitudinal vortex street in the background of non-uniform zonal wind. Accumulation of these vortices in the ionosphere medium can create the strongly turbulent state.

Keywords: Gravity waves; Shear flow; IGW transient amplification; nonlinear vortex structures

1. Introduction

In recent years an increasing interest is paid to investigation of the properties of internal gravity waves (IGW), arising as a result of vertical density stratification of the gas, and play an important role in the dynamics of both the lower and upper atmosphere and ionosphere of the earth and other planets. Grown interest, first of all, is caused primarily due to the understanding of the fact that these waves can propagate over hundreds or thousands of kilometers from the source without significant attenuation. Propagating with group velocity the IGW provide an efficient transfer of energy, heat and momentum from the troposphere into the upper atmosphere (which exceeds even the energy supplied by the solar wind), where they influence on the thermal and dynamic regimes (Francis, 1975; Kim and Mahrt, 1992; Nakamura et al., 1993; Rishbeth

and Fukao, 1995; Fritts et al., 2006; Alexander et al., 2008; Hecht et al., 2009; Alexander, 2010). Latest numerical experiments (Gavrilov and Fukao, 2001; Alexander and Rosenlof, 2003; Alexander et al., 2010) show that an adequate description of climate change and the circulation of the middle atmosphere requires taking into account the accelerations of the background flows and heat inflows generated by the waves (especially by IGW) propagating from the troposphere.

Numerous theoretical and experimental studies have established that the source of IGW motions in the atmosphere and ionosphere can be: an earthquake (Liperovsky et al, 1992; Hayakawa, 1999), volcanic eruption (Cheng, Huang, 1991), magnetic and sea storms (Testud , 1970; Golitsyn et al, 1975), hurricanes, typhoons, tornadoes, (Kuester et al., 2008; Ming et al., 2010), the solar eclipse (Chimonas and Hines, 1971), jet flows (Bertin, et al. , 1978), the terminator (Burmaka et al, 2003), spans of meteors (Pokhotelov et al., 1995), launching of powerful rockets (Burmaka et al, 2003), the polar and equatorial current systems (Chimonas and Hines, 1970), as well as industrial, military and nuclear explosions of big strength (Tolstoy and Herron, 1970; Drobjev et al, 1986; Shaefer et al., 1999)).

One of the important properties of IGW is their significant influence on the distribution of the electromagnetic waves in atmospheric-ionosphere layers (Rastogi, 1981; Gershman, 1974). Consequently, ionosphere electric currents and electromagnetic fields may re-influence the wave properties of IGW at ionosphere altitudes. In the ionosphere, in contrary to the lower layers of the atmosphere, investigating the dynamics of wave processes non-uniform and non-stationary properties of the wind process, the turbulent state of the lower ionosphere and the influence of non-uniform electromagnetic forces should be taken into account. These factors, which are due to the low density medium in the ionosphere and the relatively high conductivity of the ionosphere gas, are strongly pronounced and they can sufficiently affect the propagation characteristics of wave patterns. Consequently, the general circulation in the ionosphere must have specific features that are absent in the troposphere.

The stationary problem of the existence of ionosphere wave disturbances in case of rectilinear uniform medium flow (for large-scale Rossby type waves) has been discussed for the first time in the work of Dokuchaev (1959). It has been revealed that in the theoretical study and interpretation of the dynamics of the winds above 100 km it is necessary to consider the possible deviations from the geostrophic winds associated with the action of electromagnetic forces. Further, a number of other works have appeared (Hines and Reddy, 1967; Aburdjania and Khantadze, 2002; Aburjania et al., 2005; Aburjania et al., 2006 and others), which studied the non-stationary evolution of wind structure in the conducting ionosphere medium under the influence of the spatially non-uniform geomagnetic field.

The action of the geomagnetic field, on the one hand, leads to the inductive damping of the waves associated with Pedersen or transverse (with respect to the geomagnetic field) conductivity, on the other one - to the gyroscopic effect due to the Hall conductivity of the ionosphere acting on the perturbation like the Coriolis force. As a result of the joint action of spatially non-uniform Coriolis and electrodynamic (related to the geomagnetic field) forces the new type waves with different characteristics from the usual waves in the neutral medium may exist in the ionosphere. These waves can be called as magnetized waves.

The results of long-term observations (Gossard and Hook, 1975; Kazimirovskii and Kokourov, 1979; Pedloski, 1979) also show that at the atmospheric-ionospheric layers the spatially non-uniform zonal winds - the shear flows are permanently present, produced by nonuniform heating of the atmospheric layers by the solar radiation. In this context the problem of the generation and evolution of ordinary and magnetized waves at different layers of the atmosphere during their interaction with non-uniform zonal wind (shear flow) becomes urgent.

Interest to the shear flows, in general, is due to their widespread implementation in the near-Earth space (as noted above), astrophysical objects (galaxies, stars, jet emissions, oceans, etc.), and in the laboratory and technical equipments (pipelines, gas pipelines in the plasma magnetic traps, magnetohydrodynamic generators, etc.). The shear velocity represents a powerful source of various energy-consuming processes in the continuum. Canonical (modal) approximation of the linear wave processes - the spectral decomposition of the perturbations according to time with further analysis of Eigen values in the shear flows looses from the sight very important physical processes such as: transient amplification and mutual transformation of the linear wave modes (Reddy et al., 1993; Trefenthen et al., 1993).

A rigorous mathematical description of the specifics of the shear flows found out (Reddy et al., 1993) that at the canonical (modal) analysis of the linear processes the operators in the dynamic equations are not self-adjoint (Trefenthen et al., 1993) and, consequently, the eigen functions do not create orthogonal system, they hardly interfere with each other. This circumstance, for correct description of the phenomena, makes it necessary to estimate the results of interference of the eigen functions, which sometimes presents a huge problem.

There is another approach - so-called non-modal analysis of linear processes in the shear flows. In this approach a modified initial value problem (Cauchy problem) is solved by tracing the temporal evolution of spatial Fourier harmonics (SFH) perturbations without any spectral expansion in time (Graik and Criminale, 1986; Chagelishvili et al., 1996). Being the optimal language, the non-modal approximation greatly simplifies the mathematical description of the linear dynamics of disturbances in shear flows and allows identification of the key events (due to the non-orthogonality of the linear dynamics) that escaped from the view in a modal analysis.

In this paper we study the linear and nonlinear stages of evolution of IGW in shear zonal flows (winds) in different regions of the ionosphere. At the initial linear stage in the dynamic equations the perturbed hydrodynamic quantities are given by SFH, which corresponds to non-modal analysis in a moving coordinate system along the background wind. Non-modal mathematical analysis allows replacement of the spatial non-uniform nature of the perturbed quantities, associated with the basic zonal flow, by temporal one in the basic equations and trace the evolution of SFH disturbances according to time.

Currently, the results of numerous observations and experiments reveal the wave motion in a wide range of frequencies from the acoustic to the planetary ones in the atmosphere-ionosphere environment on almost all altitudes. In atmospheric acoustics the focus is laid on the study of internal gravity waves (IGW), representing fluctuations of atmospheric and ionospheric layers, the nature of which is mostly determined by gravity force. These oscillations are going with the frequency, at which the wave speed is comparable with the acceleration of gravity force. Therefore, for definiteness, we assume that their periods range from 5 minutes to 3 hours, and the wavelengths - from 100 m to 10 km.

In this paper, a property of internal gravity waves presents particular interest to us - propagating vertically up quite easily in an isothermal atmosphere, IGW tends to increase the amplitude of the hydrodynamic velocity exponentially with height, which follows from the conservation of energy when the density of the medium decreases with height growth (Hines, 1960; Gossard and Hook, 1975). Thus, even for the waves, the initial amplitude of which is small, the nonlinear effects at sufficiently high altitude becomes significant and must be taken into account. Indeed, it is clear that this growth can not be continued indefinitely. At some heights velocity becomes so large that the nonlinear effects can join the game. These effects stop the growth of the oscillation amplitude through the nonlinear interaction between the modes, the perturbations' energy redistribution (saturation of the waves) and, for example, self-organization of IGW vortex structures (Aburdjania, 1996, 2006). Nonlinear vortex structures transfer the trapped particles of the medium. Reaching the critical heights, the IGW structures, interacting with each other and medium, may form the atmospheric turbulence (Waterscheid and

Schubert, 1990), that creates real threats to aviation safety, but also leads to a mix of chemicals, released from the lower atmosphere, chemical reactions between them and the formation of potentially harmful compounds (Friedrich et al., 2009). Therefore, the IGW structures may also influence the formation of "space weather" by generating irregularities in the ionosphere (Schunk and Sojka, 1996).
The aim of this paper is theoretical investigation of the peculiarities of generation; intensification and further nonlinear stage of evolution of IGW structures due to the presence of local inhomogeneous zonal wind (shear flow). Section 2 explains the model of the medium and basic hydrodynamic equations for the lower ionosphere. In Sec. 3 we briefly outline the main principles of non-modal mathematical analysis and simulation results of the generation and intensification of magnetized IGW in the linear stage. In section 4, a model of the nonlinear hydrodynamic equations for the lower ionosphere is displayed, which describes the interaction of magnetized IGW structures with a shear flow. In Sec. 5 we examine the issue of the stability of the waves in shear flow and derive a necessary condition for instability. Generation mechanism of nonlinear vortex structures by non-uniform zonal wind is analyzed in Section 6. In Section 7 we study the characteristics of energy transfer by the IGW structures in the dissipative ionosphere with the shear flow. Discussion of the results is carried out in Section 8.

2. Model of environment and initial dynamic equation

Let's introduce a local system of Cartesian coordinates x, y, z with the axis x directed to the east, y axis − to the north and the z axis −vertically. We are interested in low-frequency wave motions in the ionosphere medium (consisting of electrons, ions and neutral particles) with $\omega << kc_s$ (where ω and k − the characteristic frequency and wave number of perturbation, respectively; $c_s = (\gamma P_0 / \rho_0)^{1/2}$ −the speed of sound, $\gamma = c_p / c_V$ −the ratio of specific heats, P_0 −the equilibrium gas-kinetic pressure, ρ_0 −the equilibrium density of the medium) with a horizontal spatial scale L_h of order of 10km, the vertical scale L_v is much smaller than the scale height H ($L_v << H = d\ln\rho_0 dz = c_s^2/(\gamma g)$) and the time scale τ of the order of 5 minutes $\leq \tau <$ 3 hours. Herewith, the dynamic properties of this medium, and movements of the large step anew are determined by a neutral component, because of the condition $N_{e,i} / N_n << 1$ (where $N_e, N_i = N$ and N_n - the concentration of electrons, ions and neutral components, respectively). The presence of charged particles causes the electrical conductivity of the medium and the appearance of the electromagnetic Ampere force.
For considered class of perturbations the effective magnetic Reynolds number is relatively small $R_{eff} \approx \mu_0 \sigma_{eff} V \cdot L << 1$ (where μ_0 is the permeability of free space, σ_{eff} is the effective conductivity of the ionosphere, V and L - characteristic values of velocity and perturbations, respectively), which is quite well done almost right up to F-layer of the ionosphere (Gershman, 1974; Dokuchaev, 1959; Kamide and Chian, 2007). Consequently, for the lower ionosphere, we can neglect the induced magnetic field $\mathbf{b} \approx R_{eff}\,\mathbf{B}$ and the vortical electric field $E_v \sim R_{eff}(VB)$ that arise by virtue of variation of $\mathbf{b}$. Consequently, for the class of wave perturbations the magnetic field can be assumed given and equal to the external, spatially non-uniform geomagnetic field $\mathbf{B_0}(\mathbf{B} = \mathbf{b} + \mathbf{B_0} \approx \mathbf{B_0}, \mathbf{E_v} \to 0)$. It satisfies the equation $\mathrm{div}\mathbf{B}_0 = 0$, $\mathrm{rot}\mathbf{B}_0 = 0$. At such induction free approximation consideration only of the current $\mathbf{j}$ is sufficient, arisen in the medium, ignoring the magnetic field generated by this current. In this case, the effect of geomagnetic field $\mathbf{B}_0$ on the induced current $\mathbf{j}$ in the ionosphere

plasma leads to consideration of the electromagnetic Ampere force $[\mathbf{j}\times\mathbf{B}_0]$ in the known equations of the dynamics of the ionosphere (in addition to forces: pressure, Coriolis and viscous frictions). This force causes the inductive damping (due to Pedersen currents) in the ionosphere of the earth, not less significant than usually viscous damping, especially in **F** region (Gershman, 1974; Dokuchaev, 1959).
Based on the above discussion, the basic properties of internal gravity waves in the ionosphere is advisable to consider as the initial equation that for two-dimensional motion in the plane (x, z) ($\partial/\partial y = 0$; with velocity $\mathbf{V}(V_x, 0, V_z)$, where it's assumed the acceleration to be defined due to the gravity acceleration, pressure gradient, Coriolis forces, the volumetric electrodynamic and viscous frictions (Gershman, 1974; Dokuchaev, 1959; Gossard and Hook, 1975).

$$\frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}=-\frac{\nabla\mathbf{P}}{\rho}+\mathbf{g}-2\left[\mathbf{\Omega}_0\times\mathbf{V}\right]+\frac{1}{\rho}\left[\mathbf{j}\times\mathbf{B}_0\right]+\nu\Delta\mathbf{V}, \tag{1}$$

For exclusion of high-frequency acoustic modes, let's use the condition of incompressibility of medium

$$\nabla\cdot\mathbf{V}=0. \tag{2}$$

Then, the continuity equation can be chosen as the equation of medium density in the form:

$$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+(\mathbf{V}\nabla)\rho=0 \tag{3}$$

and the medium state equation:

$$\frac{\partial P}{\partial t}+(\mathbf{V}\nabla)P=0. \tag{4}$$

Here, as usual, $\rho = N_n M = \rho_0(z)+\rho'(x,z,t)$ is the density, $P = P_0(z)+P'(x,z,t)$ −pressure, $\mathbf{g}=-g\,\mathbf{e_z}$ − the gravity acceleration; $\mathbf{e_z}$ − the unit vector along the vertical direction, i.e. along the axis z. Variables with index zero mean atmospheric parameters in the unperturbed state, and values with a prime − the disturbed ones (hereinafter, for simplicity, the primes are omitted from the quantities). M is a mass of the ion and neutral particles (molecules), ν - kinematic viscosity, $\Delta=\partial^2/\partial x^2+\partial^2/\partial z^2$ − two-dimensional Laplacian. Electromagnetic force $[\mathbf{j}\times\mathbf{B}_0]$ largely determines the specificity of ionosphere motions (Aburjania et al., 2006). Induced current density is determined from the generalized Ohm's law for the ionosphere (Gershman, 1974):

$$\mathbf{j}=\sigma_{\|}\mathbf{E}_{d\|}+\sigma_{\perp}\mathbf{E}_{d\perp}+\frac{\sigma_H}{B_0}[\mathbf{B}_0\times\mathbf{E}_d], \tag{5}$$

where the parallel $\sigma_{\|}$ (in the direction of the magnetic field $\mathbf{B}_0$), Pedersen or transverse σ_P (transverse to $\mathbf{B}_0$) and the Hall conductivities σ_H are determined by the following expressions

$$\begin{aligned}
\sigma_{\|}&=e^2N\left(\frac{1}{m\nu_e}+\frac{1}{M\nu_{in}}\right),\\
\sigma_P&=e^2N\left\{\frac{\nu_e}{m\left(\nu_e^2+\omega_{Be}^2\right)}+\frac{\nu_{in}}{M\left(\nu_{in}^2+\omega_{Bi}^2\right)}\right\},\\
\sigma_H&=e^2N\left\{\frac{\omega_{Be}}{m\left(\nu_e^2+\omega_{Be}^2\right)}-\frac{\omega_{Bi}}{M\left(\nu_{in}^2+\omega_{Bi}^2\right)}\right\},
\end{aligned} \tag{6}$$

where e, m, $\nu_e = \nu_{ei} + \nu_{en}$, $\omega_{Be} = eB_0/m$ are charge, mass, frequency of collisions between electrons and ions and neutral molecules and electron cyclotron frequency, respectively; ν_{in} and $\omega_{Bi} = eB_0/M$ the corresponding values for the ions. Assuming the ionosphere to be quasi-neutral with a high degree of accuracy, we have neglected the electrostatic $\mathbf{E}_e = -\nabla\Phi$ (Φ - electrostatic potential) and vortex parts $\mathbf{E}_v$ of the electric field. Thus, in Eq. (5) the electric field strength, taking into account the medium motion, is determined only by dynamo - field (Gershman, 1974; Dokuchaev, 1959)

$$\mathbf{E}_d = [\mathbf{V} \times \mathbf{B}_0]. \tag{7}$$

Geomagnetic field $\mathbf{B}_0(B_{0x}, B_{0y}, B_{0z})$ is considered to be dipole, which in chosen coordinate system has the following components (Dokuchaev, 1959)

$$B_{0x} = 0, \;\; B_{0y} = -B_e \sin\theta', \;\; B_{0z} = -2B_e \cos\theta', \tag{8}$$

where $B_e \approx 3{,}5 \times 10^{-5}$ Tesla (T) is a value of the geomagnetic field induction at the equator. In this case, the full geomagnetic field induction is $B_0 = B_e\left(1 + 3\cos^2\theta'\right)^{1/2}$ and $\theta' = \pi/2 - \varphi'$, φ' – geomagnetic latitude. In the same coordinate system for the components of the angular velocity of the Earth rotation $\mathbf{\Omega}_0\left(\Omega_{ox}, \Omega_{oy}, \Omega_{oz}\right)$ it can be written

$$\Omega_{0x} = 0, \quad \Omega_{0y} = \Omega_0 \sin\theta, \quad \Omega_{0z} = \Omega_0 \cos\theta \tag{9}$$

Further it's assumed that the geographic $\phi = \pi/2 - \theta$ and geomagnetic φ' latitudes are coinciding and the perturbation is located near latitude $\varphi_0 = \pi/2 - \theta_0$.

The equilibrium density of the medium is stratified due to gravitational forces. Therefore, in the thermosphere, the equilibrium density ρ_0 varies exponentially according to altitude

$$\rho_0(z) = \rho(0)\exp\left(-\frac{z}{H}\right) \tag{10}$$

For definiteness, we will consider ionosphere E-region, which is located at altitudes of $80-150$ kilometers from the Earth's surface. In this region the equilibrium parameters of the medium have the following hierarchy: $\nu_e \approx \nu_{en}$; $\omega_{Be}\omega_{Bi} >> \nu_{in}\nu_{en}$ and $\nu_{in} >> \omega_{Bi}$, which allows simplification of the expression for the induced current (5). Herewith, the condition $\nu_{in} >> \omega_{Bi}$ means that the ions are un-magnetized and their speed across the geomagnetic field coincides with the velocity of the neutrals (Aburjania et al., 2005), i.e. ions are completely entrained by the neutral ionospheric winds. However, the electrons are magnetized $\omega_{Be} >> \nu_{en}$, and they are frozen in the geomagnetic field. In this case, the Hall $\sigma_H = en/B_0$ and Pedersen σ_P conductivities are subject of the following inequality $\sigma_P \approx \sigma_H\omega_{Bi}/\nu_{in} << \sigma_H$ (Aburjania et al., 2005). For numerical calculations we use typical values of the medium parameters (Gershman, 1974; Ginzburg and Rukhadze, 1975): $N/N_n \sim 10^{-8} - 10^{-6}$, $\nu_{ei} \sim 10^3 c^{-1}$, $\nu_{en} \sim 10^4 c^{-1}$, $\nu_{in} = 10^3 c^{-1}$, $\nu_{en} \sim 10^4 c^{-1}$, $\omega_{Be} \sim 10^7 c^{-1}$, $\omega_{Bi} \sim 10^2 s^{-1}$, $\sigma_H \approx 3\times10^{-4} S/m$ and $\sigma_P \approx 10^{-4} S/m$. In the equation of the ionosphere motion (1) the part of the contribution of the Lorentz force eNB_0/ρ_0 is associated with the Hall currents and the total contribution of the Coriolis force $2\Omega_0$ has the same order $\sim 10^{-4} s^{-1}$. In addition, we take into consideration that the ratio N/ρ_0 does not depend on the vertical coordinate (height) z (Gershman, 1974). Herewith, the characteristic frequency of IGW ($\omega \sim 10^{-2} c^{-1}$) is significantly higher than the Coriolis and Hall's gyroscopic frequencies. Based on these estimations, we can conclude that the contributions of the full Coriolis and

Lorentz forces, associated with the Hall currents, have negligible impact on the dynamic properties of IGW. However, inductive damping, stipulated by Pedersen conductivity (especially in the F-region), as well as viscous damping, can not be neglected how small they can be. In investigation of the dynamics of wave disturbances in shear flows they are important as a way of redistribution of energy of the system that provides sustainable self-maintaining of the nonlinear solitary structures in the medium.
Further, the motion equation can still be more simplified if we consider the fact, that perturbation of the medium density by internal gravity waves does not exceed $3-4\%$ (Gossard and Hook, 1975; Gill, 1982). Accordingly, ratio of the perturbed density with the unperturbed one has the order $\rho'/\rho_0 \sim (1-4)\times 10^{-2}$. Based on the aforementioned, in the initial equation of motion (1) we can neglect ρ' in comparison with $\rho_0(z)$ before the inertial, Coriolis and viscous terms and using the Boussinesq approximation, we obtain the following motion equation:

$$\rho_0(z)\left(\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}\right)=-\nabla P+\rho\,\mathbf{g}-\sigma_P B_0^2\left(\mathbf{V}-B_0\frac{(\mathbf{V}\cdot B_0)}{B_0^2}\right)+\rho_0(z)\nu\Delta\mathbf{V} \qquad (11)$$

The system of equations (3), (4) and (11) presents the initial closed system of equations for both linear and nonlinear dynamics of IGW in their interaction with the geomagnetic field in the dissipative ionosphere (D, E, and F -regions).

3. Generation and intensification of IGW at linear stage of evolution

To study the linear stage of interaction of internal gravity waves with the local non-uniform zonal wind and geomagnetic field, let's linearize the system of equations (3), (4) and (11) on the background of a plane zonal shear flow (wind), which has the velocity $\mathbf{V}_0(z)$, non-uniform along the vertical: $\mathbf{V}=\mathbf{V}_0(z)+\mathbf{V}(x,z,t)$, $\rho=\rho_0(z)+\rho(x,z,t)$, $P=P_0(z)+P(x,z,t)$. Here $\mathbf{V}_0(z)$ is the background zonal wind velocity which for the vertical shear flow is given as:

$$\mathbf{V}_0(z)=v_0(z)\,\mathbf{e_x}=A\cdot z\cdot\mathbf{e_x}, \qquad (12)$$

where $A>0$ - constant parameter of the wind shear, $\mathbf{e_x}$ - a unit vector directed along the axis x.
In the selected local rectangular coordinate system for the components (11), (2), (3) and (4) we obtain the following system of linear equations:

$$\rho_0\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)V_x=-\frac{\partial P}{\partial x}-\rho_0 v_0'(z)V_z-\sigma_P B_0^2 V_x+\rho_0\nu\Delta_\perp V_x, \qquad (13)$$

$$\rho_0\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)V_z=-\frac{\partial P}{\partial z}-\rho_0 g-\sigma_P B_y^2 V_z+\rho_0\nu\Delta_\perp V_z, \qquad (14)$$

$$\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)\rho=-\frac{d\rho_0}{dz}V_z, \qquad (15)$$

$$\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)P=-\frac{dP_0}{dz}V_z, \qquad (16)$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_z}{\partial z}=0. \qquad (17)$$

Here $v_0'(z)=dv_0(z)/dz$. In this system of five equations (13) - (17) any four of them creates a closed system. To facilitate further research, we choose equation (13), (14), (15) and (17) as a closed system.

3.1. *The local dispersion equation*

The system of equations (13) - (17) presents partial differential equations with variable coefficients, depending on the spatial coordinate z. Therefore, to analyze the existence of nontrivial solutions at least at the initial stage of the evolution of wave disturbances, a local approximation is necessary, when the coefficients of equations (13) - (17) can be assumed locally uniform (constant). Then, for analyzes of the spectral characteristics, described by these equations of the disturbances, the Fourier expansion should be performed according to the spatial and temporal variables (Mikhailovskii, 1974). In Consequence to the exponential fall of the equilibrium density of the medium (10) with height, we seek a solution of equations (13) - (17) in the form of plane waves (Hines, 1960; Golitsyn, 1965; Gossard and Hook, 1975):

$$V_{x,z}(x,z,t)=\int V_{x,z}(k_x,k_z)\exp\{i[k_x x+(k_z-i/2H)z-\omega t]\}dk_x dk_z,$$
$$(P,\rho)(x,z,t)=\int (P,\rho)(k_x,k_z)\exp\{i[k_x x+(k_z+i/2H)z-\omega t]\}dk_x dk_z, \qquad (18)$$

where the spatial Fourier expansion of the wave disturbances is carried out; $\mathbf{k}\,(k_x,0,k_z)$ — wave vector and $\omega\,(k_x,k_z)$ - frequency of the waves. Inserting (18) into equations (13) - (15) and (17), we obtain the following dispersion equation:

$$(\omega-k_x v_0)^2-\frac{k_x^2}{K^2}\omega_g^2+i\frac{(\omega-k_x v_0)}{K^2}\left[k_x^2\left(\frac{\sigma_P B_{0y}^2}{\rho_0}+\nu K_1^2\right)-\right.$$
$$\left.k_x\left(k_z+\frac{i}{2H}\right)v_0'+\left(k_z^2+\frac{1}{4H^2}\right)\left(\frac{\sigma_P B_0^2}{\rho_0}+\nu K_1^2\right)\right]=0. \qquad (19)$$

Here, we introduce the notation: $\omega_g=(g/H)^{1/2}>0$ — frequency of Brunt-Vaisala for stably stratified incompressible isothermal atmosphere; $K^2=k_x^2+k_z^2+1/(4H^2)$, $K_1^2=K_2^2-ik_z/H$, $K_2^2=k_x^2+k_z^2-1/(4H^2)$, . Assuming the wave number K to be real and frequency $\omega=\omega_0+i\gamma$, $|\gamma|<<\omega_0$ to be complex, from (19) we get the expressions for the spectrum of linear fluctuations

$$\frac{\omega_0}{k_x}=v_0-\frac{v_0'}{4K^2H^2}\pm\frac{\omega_g}{K}\sqrt{1+\frac{v_0'^2}{16K^2H^2\omega_g^2}}, \qquad (20)$$

and decrement (increment) of the perturbations

$$\gamma=-\frac{k_x^2\left(\frac{\sigma_P B_{0y}^2}{\rho_0}+\nu K_2^2\right)+\left(k_z^2+\frac{1}{4H^2}\right)\left(\frac{\sigma_P B_0^2}{\rho_0}+\nu K_2^2\right)-k_x k_z v_0'}{2K^2\left[1+\frac{k_x v_0'}{4K^2H(\omega_0-k_x v_0)}\right]} \qquad (21)$$

In the absence of shear flow the formula (20) transforms into the expression for the frequency of ordinary internal gravity waves (Golitsyn, 1965):

$$\omega_0=\pm\frac{k_x\omega_g}{\left(k_x^2+k_z^2+1/(4H^2)\right)^{1/2}}. \qquad (22)$$

Formula (21) expresses the damping decrement of IGW due to induction (Pedersen) and viscous damping in the ionosphere medium:

$$\gamma = -\frac{k_x^2\left(\dfrac{\sigma_P B_{0y}^2}{\rho_0} + \nu K_2^2\right) + \left(k_z^2 + \dfrac{1}{4H^2}\right)\left(\dfrac{\sigma_P B_0^2}{\rho_0} + \nu K_2^2\right)}{2K^2}, \tag{23}$$

According to (22), phase velocity of linear IGW is in the range:

$$-V_{max} \le V_p \le V_{max}, \tag{24}$$

where $V_{max} = 2H\omega_g = 2(gH)^{1/2}$ in incompressible atmosphere. IGW is a low-frequency branch of acoustic-gravity waves (AGW), occupying an intermediate position between the frequency of inertial oscillations $\omega_i = 2\Omega_0$ and the frequency of the Brunt-Vaisala for stably stratified incompressible isothermal atmosphere ω_g, $\omega_i < \omega_0 < \omega_g$ (Gershman, 1974; Gossard and Hook, 1975). For the height of the uniform atmosphere $H \approx 4.5 \div 6$ km, we can estimate the value of maximal phase velocity of linear IGW $V_{max} \approx 440$ m/s, the frequency $\omega_g \approx 1.7\times10^{-2}$ and $\Omega_0 \approx 10^{-4}$. So, IGW disturbances cover the following range of low-frequency oscillations $10^{-4}c^{-1} < \omega < 1.7\times10^{-2}c^{-1}$ - and can be supersonic $V_p \ge c_s \approx 330$ m/s.

Considered waves have frequency limit ω_g and gravitational effects play an important role for them, as reflected in their name - IGW. From (22), it follows that for practically important case of relatively short waves, $k_z^2 >> k_x^2$ and $k_z H >> 1$, and phase ($\mathbf{V_p} = (\omega/k^2)\,\mathbf{k}$) and group ($\mathbf{V_g} = \nabla_{\mathbf{k}}\omega$) the velocities of IGW in the windless atmosphere are given by:

$$V_{px} = \frac{\omega_g k_x^2}{k_z^3}, \quad V_{pz} = \frac{\omega_g k_x}{k_z^2}; \quad V_{gx} = \frac{\omega_g}{k_z}, \quad V_{gz} = -\frac{\omega_g k_x}{k_z^2}. \tag{25}$$

It is evident that short IGW possess strong spatial dispersion. The direction of the phase velocity is close to vertical, $|V_{pz}| >> |V_{px}|$, and the group speed is almost horizontal, $|V_{gx}| >> |V_{gz}|$. In the long-wave case $k_z H << 1$, IGW is almost dispersion-less.

Let's estimate appropriate wavelengths for IGW in the dissipative ionosphere. At ground level the kinematic molecular viscosity is determined as $\nu_0 \approx 1.3\times10^{-5} m^2/s$, at the level of the E-layer (~110 km) $-\nu_{110} \approx 1.3\times10^2 m^2/s$ and at the level of the F-layer ((250-300)km) $-\nu_{300} \approx 1.3\times10^6 m^2/s$ (Gossard and Hook, 1975). Appropriate minimal lengths of IGW with period 10 minute in the presence of kinematic viscosity for near Earth regions is $\lambda_0 \sim 10^{-1}$ m and with the height growth will increase correspondingly an appropriate minimal IG wavelength and for E-layer it would be $\lambda_{110} \sim 10$ m, for F-layer $-\lambda_{300} \sim 10$ km (Gossard and Hook, 1975). Coming through this and taking into account that the turbulent viscosity in the low atmosphere increases the IGW scale [Gossard and Hook, 1975], further we will consider the gravitational waves, the wavelengths of which fall into the range 100 m $\le \lambda \le 10$ km.

Let us estimate the damping rate of IGW. For ground level the parameters of the medium and the waves are $\sigma_p \cong 5\times10^{-7}$ S/m, $\rho_0 = 1.3$ kg/m^3, $B_0 = 0.5\times10^{-4}$ T, $\lambda \approx 10^4$ m, $\nu_0 \approx 1.3\times10^{-5} m^2/s$ and according to (21), viscous damping decrement of IGW structures is $\gamma_{\nu,0} \approx k^2\nu \sim 5\times10^{-12}$ s^{-1}, the decrement of the induction decay $-\gamma_{\sigma,0} \approx \sigma_p B_0^2/\rho_0 \sim 10^{-15}$ s^{-1}. For E-layer the parameters of the medium and the waves are $\sigma_p \cong 3\times10^{-4}$ S/m, $\rho_0 = 10^{-10}$ kg/m^3, $B_0 = 0.5\times10^{-4}$ T, $\lambda \approx 10^4$ m,

$\nu_{110} \approx 1.3\times10^2 \mathrm{m}^2/\mathrm{s}$: $\gamma_{\nu,110} \approx k^2\nu \sim 5\times10^{-5}$ s^{-1}, $\gamma_{\sigma,110} \approx \sigma_p B_0^2/\rho_0 \sim 10^{-3}$ s^{-1}. For F-layer we use the typical values of parameters $\sigma_H \approx en/B_0 \sim 10^{-4}$ S/m, $\sigma_p \approx \sigma_H \omega_{ci}/\nu_{in} \sim \sigma_H$, $\omega_{ci} \sim 3\times10^2$ s^{-1}, $\nu_{in} \le 10$ s^{-1}, $\nu_{300} \approx 1.3\times10^6 \mathrm{m}^2/\mathrm{s}$. For damping rates we obtain correspondingly: $\gamma_{\nu,300} \sim 10^{-1}$ s^{-1}, $\gamma_{\sigma,300} \sim 10^{-3}$ s^{-1}. So, at the different levels of ionosphere the values of the viscous and induction damping of IG structures are different, and it should be considered in dynamic problems involving IGW structures.

It should be noted, that according to (20), the non-uniform zonal wind greatly expands the range of IGW in the ionosphere. Moreover, the shear flow feeds the medium with energy (see formula (21)), which is responsible for the generation-swing of IGW and development of linear shear instability with a characteristic growth rate:

$$\gamma_A \sim \frac{k_x k_z}{K^2} A. \qquad (26)$$

From (23) it's obvious, that considered ionospheric shear flow can become the source of the instability at the condition $\gamma_A \ge \gamma_\nu, \gamma_\sigma$. According to (26), for generation of the IGW structures it is necessary the shear flow velocity to have even first derivative according to the vertical coordinate, different from zero ($v_0'(z) = A \ne 0$). As it was mentioned in the works (Margetroyd, 1969; Mayer et al., 1990), the typical value of the dimensional parameter of the shear flow $(A)\,\mathrm{s}^{-1}$ for the ionospheric F-region equals $A = v_0' \approx (0.015 \div 0.15)\mathrm{s}^{-1}$ as well. Taking it into account from (26) we obtain $\gamma_A \ge 10^{-1}\mathrm{s}^{-1}$. Thus, the condition of the generation and amplification of IGW perturbations (inequality $\gamma_A \ge \gamma_\nu, \gamma_\sigma$) in the different levels of the ionosphere (especially, in D and E -regions) can be satisfied and the shear instability can be developed. This conclusion can be made by virtue of above used modal (local - spectral) approach, which can't give more information about the features of the shear flow instability. But this doesn't mean that such instability always arises and remains in such form. This is exactly due to non-adequacy of modal approach at investigation of the features of shear flows, which is already considered in the introduction. In shear flows the modal approach can detect only possibility of instability. But for investigation of instability generation conditions and its temporal development in the ionosphere an alternative approach, namely, non-modal mathematical analysis becomes necessary.

As it will be shown in the next section on the basis of more adequate method for such problems –non-modal approximation, shear flows can become unstable transiently till the condition of the strong relationship between the shear flows and wave perturbations is satisfied (Chagelishvili et al., 1996; Aburjania et al., 2006), e. i. the perturbation falls into amplification region in the wave number space. Leaving this region, e. i. when the perturbation passes to the damping region in the wave vector space, it returns an energy to the shear flow and so on (if the nonlinear processes and self-organization of the vortex structure will not develop before) (Aburjania et al., 2010). The experimental and observation data shows the same (Gossard and Hooke, 1975; Pedlosky, 1979; Gill, 1982).

Thus, non-uniform zonal wind or shear flow can generate and/or intensify the internal gravity waves in the ionosphere and provoke transient growth of amplitude, i.e. transient transport the medium into an unstable state. In the next subsection we confirm this view by using a different, more self-consistent method for the shear flow.

3.2. *Non-modal analysis of shear instability of the waves in the ionosphere*

Deriving (19) - (21) we used so-called local approximation, i.e. it is assumed that v_0, v_0', ρ_0 and P_0 are locally uniform and we have provided the Fourier expansion of the physical quantities. Local approximation has limited applicability in non-uniform environment, and especially in the shear flows

(Mikhailovskii, 1974). In particular, the results received at such approximation are applicable only for the initial stage of evolution of the perturbations. In general, if the background flow is spatially non-uniform along the vertical, then we are not to provide Fourier expansion along the axis z. According to the papers (Graik et al., 1986; Reddy et al., 1993; Trefenthen et al., 1993; Chagelishvili et al., 1996; Aburjania et al., 2006), when studying the evolution of wave disturbances in shear flows at the linear stage the non-modal mathematical analysis is better to be used than a modal approach (i.e. direct Fourier expansion). Therefore, further analysis of the features of magnetized IGW wave at the linear stage in the ionosphere should be conducted in accordance with a non-modal approach. For this purpose, the moving coordinate system $X_1O_1Y_1$ is more convenient with origin O_1 and the axis Y_1, which coincides with the same characteristics of the equilibrium local system XOY, the axis X_1 flowing along the unperturbed (background) wind. In our problem, this transformation of the coordinate system is equivalent to the following replacement of the variables:

$$x_1 = x - azt, \quad y_1 = y, \quad t_1 = t, \tag{27}$$

or

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_1} - az\frac{\partial}{\partial x_1}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial x_1}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z_1} - at_1\frac{\partial}{\partial x_1}. \tag{28}$$

With these new variables equation (13), (14), (15) and (17) take the form

$$\rho_0\frac{\partial V_x}{\partial t_1} = -\frac{\partial P}{\partial x_1} - \rho_0 v_0' V_z - \sigma_P B_0^2 V_x + \nu\rho_0\left\{\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial z_1} - at_1\frac{\partial}{\partial x_1}\right)^2\right\}V_x, \tag{29}$$

$$\rho_0\frac{\partial V_z}{\partial t_1} = -\left(\frac{\partial}{\partial z_1} - At_1\frac{\partial}{\partial x_1}\right)P - \rho_0 g - \sigma_P B_{0y}^2 V_z + \nu\rho_0\left\{\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial z_1} - at_1\frac{\partial}{\partial x_1}\right)^2\right\}V_z, \tag{30}$$

$$\frac{\partial\rho}{\partial t_1} = \frac{\rho_0}{H}V_z, \tag{31}$$

$$\frac{\partial V_x}{\partial x} + \left(\frac{\partial}{\partial z_1} - At_1\frac{\partial}{\partial x_1}\right)V_z = 0. \tag{32}$$

Coefficients of the initial system of linear equations (13) - (16) depended on the spatial coordinate z. Such mathematical transformations replace this spatial non-uniform property into temporal one (see eq. (29) - (32)). Thus, the initial-boundary problem is reduced to the initial problem of Cauchy type. Since now the coefficients of (29) - (32) are independent of spatial variables, the Fourier transformation of these equations with respect to spatial variables x_1, z_1 is already possible without any local approximation, the temporal evolution of these spatial Fourier harmonics (SFH) we consider independently:

$$\begin{Bmatrix} V_x, z(x_1, z_1, t_1) \\ \rho, P(x_1, z_1, t_1) \end{Bmatrix} = \int\int_{-\infty}^{\infty}\int dk_{x_1} dk_{z_1}\begin{Bmatrix} \tilde{V}_{x,z}(k_{x_1}, k_{z_1}, t_1) \\ \tilde{\rho}, \tilde{P}(k_{x_1}, k_{z_1}, t_1) \end{Bmatrix} \times \exp(ik_{x_1}x_1 + ik_{z_1}z_1). \tag{33}$$

Here the factors with a tilde (for example $\tilde{V}_x$) indicate spatial Fourier harmonics (SFH) of the relevant physical quantities. Inserting (33) into equations (29) - (32), and passing to dimensionless variables,

$$\tau \Rightarrow \omega_g t_1; \quad V_{x,z} \Rightarrow \frac{\tilde{V}_{x,z}}{\omega_g H}; \quad \rho \Rightarrow \frac{\tilde{\rho}}{\rho_0}; \quad P \Rightarrow \frac{-i\tilde{P}}{\rho_0\omega_g^2 H^2};$$

$$(x, z) \Rightarrow \frac{(x_1, z_1)}{H}; \quad S \Rightarrow \frac{A}{\omega_g}; \quad k_{x,z} \Rightarrow k_{x_1,z_1}H; \quad k_z = k_z(0) - k_x S\tau;$$

$$k^2(\tau)=(k_x^2+k_z^2(\tau))\,;\,\nu\Rightarrow\frac{\nu}{\omega_g H^2};\;\; b_0\Rightarrow\frac{\sigma_P B_0^2}{\rho_0\omega_g}\,;b_y\Rightarrow\frac{\sigma_P B_y^2}{\rho_0\omega_g}\,; \tag{34}$$

for each SFH perturbed quantities, we obtain

$$\frac{\partial V_x}{\partial\tau}=-SV_z+k_xP-[b_0+\nu k^2(\tau)]V_x\,, \tag{35}$$

$$\frac{\partial V_z}{\partial\tau}=k_z(\tau)P-\rho-[b_y+\nu k^2(\tau)]V_z\,, \tag{36}$$

$$\frac{\partial\rho}{\partial\tau}=V_z\,, \tag{37}$$

$$k_xV_x+k_z(\tau)V_z=0\,. \tag{38}$$

Closed system of equations (35) - (38) describes the linear interaction of IGW with a shear flow and the evolution of the generated disturbances in the dissipative ionosphere medium. We note once again that after these transformations the wave vector $k\,(k_x,k_z(\tau))$ of the perturbation became dependent on time: $k_z(\tau)=k_z(0)-k_xS\cdot\tau$; $k^2(\tau)=(k_x^2+k_z^2(\tau))$. Variation of the wave vector according to time (i.e. splitting of the disturbances' scales in the linear stage) leads to significant interaction in the medium even of such perturbations, the characteristic scale of which are very different from each other at the initial time (Aburjania et al., 2006).

On the basis of (35) - (37) an equation of energy transfer of the considered wave structures can be obtained, which gives possibility to identify the pattern of energy density variation with time:

$$\frac{dE(\tau)}{d\tau}=-\frac{S}{2}\Big(V_x^*(\tau)\cdot V_z(\tau)+V_x(\tau)\cdot V_z^*(\tau)\Big)-b_1(\tau)|V_x|^2-b_2(\tau)|V_z|^2, \tag{39}$$

Here the asterisk denotes the complex conjugate values of the indignations, $b_1(\tau)=b_0+\nu k^2(\tau)$, $b_2(\tau)=b_y+\nu k^2(\tau)$ and the density of the total dimensionless energy of the wave perturbations $E(\tau)$ in the wave number space is given by:

$$E[k(\tau)]=\frac{1}{2}\Big(|V_x|^2+|V_z|^2+|\rho|^2\Big). \tag{40}$$

It's obvious that the transient evolution of wave energy structures in the ionosphere is due to the shear flow $(S\neq 0,\;\; A\neq 0)$, dissipative processes - induction decay ($b_0\neq 0,\;\; b_y\neq 0$) and viscosity ($\nu\neq 0$). In the absence of shear flow $(S=0,\;\; A=0)$, and dissipative processes $(\nu=0,\;\sigma_P=0)$, the energy of the considered wave disturbances in the ionosphere conserves $dE(\tau)/d\tau=0$. The total energy density of the perturbations (40) consists of two parts: $E[k]=E_k+E_t$, where the first term is the kinetic energy of perturbation $E_k=\Big(\Big\||V_x|^2+|V_z|^2\Big\|\Big)/2$, and the second - thermobaric energy $E_t=|\rho|^2/2$, stipulated due to the elasticity of perturbations.

To emphasize the pure effect of shear flow on the evolution of IGW, for simplicity, we consider non-dissipative ionosphere, i.e. we suppose that $(\nu=0,\;\sigma_P=0)$. Further, we determine on the basis of equation (39) what actually leads the evolution of the energy of the wave disturbance to – does their energy increase or decrease? To answer we must calculate the right-hand side part of equation (39). For this purpose we must find the solutions of equations (35) - (38) at $b_1=b_2=0$. Differentiating (36) with respect to time and using (35) (37) and (38), we obtain the second-order equation for the vertical velocity components:

$$\frac{d^2V_z}{d\tau^2}+R_1(\tau)\frac{dV_z}{d\tau}+R_2(\tau)V_z=0, \tag{41}$$

where

$$R_1(\tau)=-4Sk_x\frac{k_z(\tau)}{k^2(\tau)},\quad R_2(\tau)=(2S^2+1)\frac{k_x^2(\tau)}{k^2(\tau)}. \tag{42}$$

Equation (41) can be simplified by introducing a new variable (Magnus, 1976). Assuming

$$V_z=V\exp\left[-(1/2)\int R_1(\tau')d\tau'\right]. \tag{43}$$

Let's transform (41) to the equation of a linear oscillator with time dependent parameters:

$$\ddot{V}+\Omega^2(\tau)V=0, \tag{44}$$

where

$$\ddot{V}=\frac{d^2V}{d\tau^2};\quad \Omega^2(\tau)=R_2(\tau)-\frac{1}{2}\dot{R}_1(\tau)-\frac{1}{4}R_1^2(\tau)=\frac{k_x^2}{k^2(\tau)}. \tag{45}$$

The equation (44) is well known in mathematical physics. This is an equation of linear oscillations of a mathematical pendulum, length of which changes. The value $\Omega(\tau)$ determines the angular velocity of the pendulum.

We solve the equation (44) in the adiabatic approximation (Zeldovich and Mishkis, 1972), i.e. when dependence of $\Omega(\tau)$ on time is adiabatically slow:

$$\left|\dot{\Omega}(\tau)\right| << \Omega^2(\tau). \tag{46}$$

Taking into account the definition of the parameter $\Omega(\tau)$ the equation (46) can be rewritten as

$$S\cdot\left|k_z(\tau)\right| << \left[k_x^2+k_z^2(\tau)\right]^{1/2}. \tag{47}$$

For the real ionospheric shear flow $S << 1$ (see definition (34)), so it can be said that the condition (47) holds for a wide range of variations of wave numbers $\left|k_z(\tau)=k_z(0)-k_xS\tau\right|$. In other words, when the temporary variation of $\left|k_z(\tau)\right|$ is due to the linear drift of the wave vector in the space of wave numbers, the condition (46) or (47) is valid in all stages of the evolution of IGW. In this case, an approximate solution of homogeneous equation (44) can be represented as:

$$V=\frac{C}{\sqrt{\Omega(\tau)}}\exp\left[i\varphi(\tau)\right], \tag{48}$$

where $C=\mathrm{const}$ and

$$\varphi(\tau)=\int_0^\tau\Omega(\tau')d\tau'=\frac{1}{S}\ln\left|\frac{k_z(0)+k(0)}{k_z(\tau)+k(\tau)}\right|.$$

Substituting (48) in (43), and then - into the equations (35) - (38), we can finally construct the solutions for physical quantities:

$$V_z(\tau)=\frac{V_z(0)\cdot k^2(0)}{k_x^{1/2}\cdot k^{3/2}(\tau)}\exp\left[i\varphi(\tau)\right], \tag{49}$$

$$V_x(\tau)=-\frac{V_x(0)\cdot k_z(\tau)\cdot k^2(0)}{k_x^{3/2}\cdot k^{3/2}(\tau)}\exp\left[i\varphi(\tau)\right], \tag{50}$$

$$\rho(\tau)=-i\frac{\rho(0)\cdot k^2(0)}{k_x^{3/2}\cdot k^{1/2}(\tau)}\exp\left[i\varphi(\tau)\right], \tag{51}$$

$$P(\tau) = \frac{P(0)\cdot k^2(0)}{k_x^{3/2}\cdot k^{5/2}(\tau)}\left[2Sk(\tau) - ik_z(\tau)\right]\exp\left[i\varphi(\tau)\right], \tag{52}$$

$$k_x V_x(0) + k_z(0)V_z(0) = 0\,. \tag{53}$$

Here, in the expressions (49) - (53) for the values of physical quantities are considered the real parts. Substituting (49) - (51) into equations (39) and (40), we obtain an expression for the normalized energy density of the Fourier harmonics:

$$\overline{E}(\tau) = \frac{E(\tau)}{E(0)} = \frac{\left(1+k_0^2\right)^2}{\left[1+(k_0 - S\tau)^2\right]^{1/2}}, \tag{54}$$

and for the IGW energy transport equation (at $b_1 = b_2 = 0$)

$$\frac{d\overline{E}(\tau)}{d\tau} = \frac{\left(1+k_0^2\right)^2\cdot\left(k_0 - S\tau\right)}{\left[1+\left(k_0 - S\tau\right)^2\right]^{3/2}}. \tag{55}$$

Here for the convenience of numerical analysis a new parameter $k_0 = k_z(0)/k_x$ is introduced.

Using equations (54) and (55) we can determine an expression for the increment (decrement) of the shear instability $\Gamma(\tau) = (1/\overline{E}(\tau))\cdot d\overline{E}(\tau)/d\tau$ in the non-dissipative ionosphere:

$$\Gamma(\tau) = \frac{k_0 - S\tau}{1+(k_0 - S\tau)^2}. \tag{56}$$

Evolution of the dimensionless normalized energy density of the SFH (54) and the increment of shear instability (56) are presented in Figures 1,2. In the initial stage of evolution when $k_0 = k_z(0)/k_x > 0$ (when $k_z(\tau) > 0$) over time τ, $0 < \tau < \tau^* = k_z(0)/(Sk_x)$, the denominator (54) decreases and, accordingly, the energy density of IGW increases monotonically and reaches its maximum value (exceeding its initial value by an order) at the time $\tau = \tau^*$. Further, at $\tau^* < \tau < \infty$ the energy density begins to decrease (when $k_z(\tau) < 0$), and monotonically returns to its initial approximately constant value. In other words, in the early stages of evolution, temporarily, when $k_z(\tau) > 0$ and IGW perturbations are in the intensification region in wave-number space, the disturbances draw energy from the shear flow and increase own amplitude and energy by an order during the period of time $0 < \tau < \tau^* = k_z(0)/(Sk_x) = 100$. Then (if the nonlinear processes and the self-organization of the wave structures are not turned on), when $k_z(\tau) < 0$, IGW perturbation enters the damping region in wave number space and the perturbation returns energy back to the shear flow over time $\tau^* < \tau < \infty$ (Fig. 1, 2) and so on. Such transient redistribution of energy in the medium with the shear flow is due to the fact that the wave vector of the perturbation becomes a function of time $k = k(\tau)$, i.e. disturbances' scale splitting takes place. The structures of comparable scales effectively interact and redistribute free energy between them. Taking into account the induction and viscous damping (see equation (39)) the perturbation's energy reduction in the time interval $\tau^* < \tau < \infty$ is more intensive than that shown on fig. 1, the decay curve in the region $\tau^* < \tau < \infty$ becomes more asymmetric (right-hand side curve becomes steeper), and part of the energy of the shear flow passes to the medium in the form of heat.

Thus, even in a stable stratified ionosphere ($\omega_g^2 > 0$), temporarily, during the time interval $0 < t^* \approx 100/(\omega_g) \sim 5\cdot 10^3\,\mathrm{s} \approx 1.5$ hour IGW-intensively draws energy from the shear flow and increases own energy and amplitude by an order. Accordingly, the wave activity will intensify in the given region of the ionosphere due to the shear flow (inhomogeneous wind) energy.

4. Nonlinear model dynamic equation for internal gravity waves in the ionosphere

For further analysis of the evolution of the IG wave disturbances it's necessary to construct a self-consistent simplified nonlinear dynamic equation on the basis of equations (11) and (3), that takes into account the presence of non-uniform zonal wind with velocity $\mathbf{V}_0(z) = v_0(z)\,\mathbf{e_x}$ in the ionosphere medium. With this purpose writing equation (11) for horizontal V_x and vertical V_z velocity components, and differentiating the first equation according to the coordinate z and the second equation – according to the coordinate x, subtracting the second equation from the first one, we get:

$$\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)\Delta\psi - v_0''(z)\frac{\partial\psi}{\partial x} + J(\psi,\Delta\psi) = -\frac{g}{\rho_0}\frac{\partial\rho}{\partial x} -$$
$$\frac{d\ln\rho_0}{dz}\left[\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z} - v_0'(z)\frac{\partial\psi}{\partial x} + J(\psi,\frac{\partial\psi}{\partial z})\right] -$$
$$\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\sigma_P B_0^2\right)\cdot\frac{\partial\psi}{\partial z} - \frac{\sigma_P B_0^2}{\rho_0}\cdot\frac{\partial^2\psi}{\partial z^2} - \frac{\sigma_P B_{0y}^2}{\rho_0}\cdot\frac{\partial^2\psi}{\partial x^2} + \nu\Delta^2\psi\,. \qquad (57)$$

Similarly, we transform the continuity equation (3):

$$\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)\rho + J(\psi,\rho) = -\frac{d\rho_0}{dz}\cdot\frac{\partial\psi}{\partial x}. \qquad (58)$$

Here, according to incompressibility condition of the considered two-dimensional perturbation ($\nabla\cdot\mathbf{V}=0$) (2), the stream function introduced as

$$V_x = -\frac{\partial\psi}{\partial z},\quad V_z = \frac{\partial\psi}{\partial x}, \qquad (59)$$

and the operator of the Jacobian $J(a,b) = \partial a/\partial x\cdot\partial b/\partial z - \partial a/\partial z\cdot\partial b/\partial x$.
For convenience of the further analysis, we turn to usual field variables for an isothermal atmosphere (Hines, 1960; Gossard and Hook, 1975):

$$\psi = \overline{\psi}\exp\left(\frac{z}{2H}\right), \rho = \overline{\rho}\exp\left(-\frac{z}{2H}\right). \qquad (60)$$

Substituting (60) into (57) and (58), replacing the factor by $\exp[z/(2H)]\approx 1$ before the nonlinear term (i.e. $k_z >> 1/(2H)$ - short wavelength waves due to vertical), considering that the parameter $(\sigma_P B_0^2/\rho_0)$ does not depend on the coordinate z (Gershman, 1974) and introducing a new variable $R = g\overline{\rho}/\rho_0(0)$, we obtain the following closed system of equations:

$$\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)\left(\Delta\overline{\psi} - \frac{\overline{\psi}}{4H^2}\right) + \left(\frac{v_0'(z)}{H} - v_0''(z)\right)\frac{\partial\overline{\psi}}{\partial x} + J(\overline{\psi},\Delta\overline{\psi}) = -\frac{\partial R}{\partial x} -$$
$$\frac{\sigma_P B_0^2}{\rho_0}\cdot\left(\frac{\partial^2\overline{\psi}}{\partial z^2} - \frac{\overline{\psi}}{4H^2}\right) - \frac{\sigma_P B_{0y}^2}{\rho_0}\cdot\frac{\partial^2\overline{\psi}}{\partial x^2} + \nu\Delta^2\overline{\psi}\,, \qquad (61)$$

$$\left(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\right)R + J(\overline{\psi},R) = \omega_g^2\cdot\frac{\partial\overline{\psi}}{\partial x}. \qquad (62)$$

Here $\omega_g = (g/H)^{1/2} > 0$ — frequency of Brunt-Vaisala for stably stratified incompressible isothermal atmosphere, $v_0'(z) = dv_0(z)/dz$ and $v_0''(z) = d^2v_0(z)/dz^2$.

The system of equations (61) and (62) describes the nonlinear interaction of internal gravity structures with an inhomogeneous zonal wind and the geomagnetic field in an incompressible isothermal dissipative ionosphere.

5. IGW stability criteria in the ionosphere with an inhomogeneous zonal wind

Nature of plane shear flow greatly defines the evolution of wave disturbances in the environment. Herewith, the shear flows in hydrodynamics and magneto-hydrodynamics are often unstable (Mikhailovskii, 1974; Gossard and Hook, 1975; Timofeev, 2000). The presence of the terms proportional to v_0' and v_0'' in equation (61) is related to the instability criterion (condition) of the shear flow. In the linear approximation for small perturbations of the form $(\overline{\Psi}(x,z,t), R(x,z,t)) = (\Psi_1(y), R_1(y))\exp(ik_x x - i\omega t)$, from the equations (61) and (62) follows an equation of the Orr-Sommerfeld

$$-i\left[\nu\left(\frac{d^2}{dz^2}-k_x^2\right)^2 + b_{0y}k_x^2 - b_0\left(\frac{d^2}{dz^2}-\frac{1}{4H^2}\right)\right]\Psi_1 + (\omega - k_x V_0)\left[\frac{d^2}{dz^2}-k_x^2-\frac{1}{4H^2}\right]\Psi_1 - \left[k_x\left(\frac{v_0'}{H}-v_0''\right)-\frac{k_x^2\omega_g^2}{\omega-k_x v_0}\right]\Psi_1 = 0. \quad (63)$$

Neglecting the dissipation effects (σ_P, $\nu \to 0$) from the equation (63) we get

$$\Psi_1'' - \left[k_x^2 + \frac{1}{4H^2} + \frac{k_x(v_0'/H - v_0'')}{\omega - k_x V_0} - \frac{k_x^2\omega_g^2}{(\omega-k_x v_0)^2}\right]\Psi_1 = 0, \quad (64)$$

where $\Psi_1'' = d^2\Psi_1/dz^2$. Equation (64) is a modification of the well-known Rayleigh equation (Timofeev, 2000) (at $1/H \to 0, \omega_g \to 0$). To determine the shear flow instability criterion in our case, we multiply (64) by Ψ_1^*, subtract the complex-conjugate expression from the result, and integrate the resulting expression in the borders from z_1 to z_2 of the plasma flow:

$$\int_{z_1}^{z_2}\frac{d}{dz}\left(\Psi_1^*\frac{d\Psi_1}{dz}-\Psi_1\frac{d\Psi_1^*}{dz}\right)dz - \int_{y_1}^{y_2}\left[\frac{1}{\omega-k_x v_0}-\frac{1}{\omega^*-k_x v_0}\right]k_x(v_0/H - v_0'')|\Psi_1|^2 dz -$$

$$\int_{z_1}^{z_2}\left[\frac{k_x^2\omega_g^2}{(\omega-k_x v_0)^2}-\frac{k_x^2\omega_g^2}{(\omega^*-k_x v_0)^2}\right]|\Psi_1|^2 dz = 0. \quad (65)$$

Assuming that the perturbation frequency is complex $\omega = \omega_0 + i\gamma$ (where ω_0 is eigen frequency of linear IGW), and the wave vector k_x - the real value, the imaginary part of equation (65) can be written as:

$$2\gamma\int_{z_1}^{z_2}\left[\frac{2\omega_1 k_x^2\omega_g^2}{(\omega_1^2+\gamma^2)^2}+\frac{k_x(v_0''-v_0'/H)}{\omega_1^2+\gamma^2}\right]|\Psi_1|^2 dz = 0, \quad (66)$$

where $\omega_1 = \omega_0 - k_x v_0$. In the case of ω_1, γ, $\left|\Psi_1^2\right| > 0$ from (66) the condition of linear instability of a shear flow it follows:

$$\frac{2\omega_1 k_x^2 \omega_g^2}{(\omega_1^2+\gamma^2)} + k_x\left(v_0'' - \frac{v_0'}{H}\right) = 0 . \qquad (67)$$

For a critical level of the ionosphere, where the phase velocity $V_p = \omega / k_x$ matches the speed of wind v_0, $V_p = v_0(z)$ (i.e. $\omega_1 = \omega_0 - k_x v_0 \approx 0$), equality (67) can be rewritten as:

$$v_0''(y) - v_0' / H = 0 . \qquad (68)$$

Conditions (67) and (68) can be called as modified Rayleigh condition ($v_0'' = 0$) for IGW at the appropriate parameters of the zonal flow, waves and environments. Implementation of this equation (67) (or (68)) in a resonant point $z = z_r$ of the shear flow is a necessary condition for instability.
In the Earth's atmosphere, the value v_0' / H can be both larger and smaller than v_0''. So, according to (68) the disturbance of the zonal wind may arise occasionally, such that in a critical layer $z = z_r$ the condition $|v_0''| = |v_0' / H|$ is fulfilled. This causes instability for some time, after which the zonal wind is reconstructed and becomes stable again, etc.

Let's briefly discuss the features and consequences of instability in simple shear flow of the ionosphere, where the rate of the local wind in the environment varies linearly - $v_0(z) = S \cdot z$, where $S > 0$ - constant parameter of the wind shear. For such wind velocity profile the necessary condition for the development of shear instability (67) is fulfilled at $|\omega_0| < |k_x v_0|$. In this case, according to section 3, shear instability develops even in a stable stratified ionosphere and temporarily, during the time interval $0 < t^* \approx 62{,}5/(\omega_b) \sim 3 \cdot 10^4$ IGW intensively draws energy of shear flow and increases own energy and amplitude by an order**.** Accordingly, the wave activity will intensify in this region of the ionosphere due to the energy of the shear flow (non-uniform wind).

6. Nonlinear vortex structures governed by the shear flow

As noted above, the spontaneously excited internal gravity waves at different layers of the ionosphere intensively draw energy from the shear flow at a certain point (in particular, for a time interval $0 < \tau \le \tau^*$) in their evolution. Receiving energy, amplitude of IGW increases (by an order of magnitude) and, accordingly, the nonlinear processes come into play. In this case, in the initial dynamic equation (61) and (62) the nonlinear terms can no longer be neglected and the full nonlinear system has to be investigated.
We proceed to study the influence of nonlinear effects on the dynamics of IGW structures in dissipative ionosphere. The results of the observations and targeted experiments show (Bengtsson and Lighthill, 1985; Chmyrev et al., 1991; Nezlin, 1994; Sundkvist, et al., 2005) that nonlinear solitary vortex structures can be generated at different layers of the atmosphere-ionosphere-magnetosphere. These structures transfer trapped rotating medium particles. Moreover, the ratio of the rotational speed of the particles U_c to the speed of motion of nonlinear structures U is given by $U_c / U \ge 1$ (Monin, 1978).
We introduce the characteristic time T and spatial scales L of the nonlinear structures. Using equation (11), (61) we can establish the relation between quantities: $U_c \sim V$, $U \sim L/T$. Similarly, for the ratio of the nonlinear term with the inertial one, we have: $(V\nabla)V/(\partial V/\partial t) \sim V/(L/T) \sim U_c/U$. Thus, the nonlinearity plays an essential role for wave processes satisfying the condition $U_c \ge U$. This estimation shows that nonlinear effects play a crucial role in the dynamics of IGW-type wave, the initial linear

stage of development of which is considered in previous section. Inequality $U_c \geq U$ coincides with the anti-twisting condition (Williams and Yamagata, 1984). Satisfying just the latter condition the initial nonlinear dynamic equations (61) and (62) may have the solitary (vortex) solutions (Williams, Yamagata, 1984; Nezlin, Chernikov, 1999).

From the general theory of nonlinear waves is well known the fact (Whitham, 1977) that if in the system the nonlinear effects are significant, then the principle of superposition can't be applied and the solution in the form of a plane wave is unjust. Nonlinearity distorts the wave profile and the wave form differs from a sinusoid. If in a nonlinear system the dispersion (or non-uniform equilibrium parameters of the medium) is lacked, all small-amplitude waves with different wave numbers k propagate with the same speed and have the opportunity for a long time interaction with each other. So, even a small nonlinearity leads to the accumulation of distortions. Such nonlinear distortion, as a rule, leads to the wave front curvature growth and its upset (breaking) or to the formation of the shock wave. In the presence of dispersion the phase velocities of waves with different k vary with the latter, the waves with different k propagate with different velocities and virtually unable to interact with each other. Therefore, the wave packet tends to spreading. For not very large amplitude the wave dispersion can compete with the nonlinearity. Because of this before breaking the wave may split into separate nonlinear wave packets, and the shock wave will not form. Indeed, in the real atmosphere, the shock wave, as a rule, (spontaneously, without external influence) is not formed spontaneously. Primarily, this means that in the atmosphere-ionosphere medium dispersion effects are strongly pronounced and significantly compete with nonlinear distortion. If the nonlinear steepening of the wave is exactly compensated by the dispersion spreading, there may appear the stationary waves such as solitary vortices propagating in a medium without changing its shape.

It should be noted also, that the results of ground and satellite observations show clearly that in the different layers of the ionosphere the zonal winds (currents) are permanently present, which are non-uniform along the vertical (Gershman 1974; Gossard and Hook, 1978; Kazimirovskii and Kokourov, 1979). As noted in section 3, at interaction with non-uniform zonal flow the wave disturbance obtains an additional dispersion as well as a new source of amplification and the nonlinear effects come into play in their dynamics. Thus, the ionospheric medium with shear flow creates a favorable condition for the formation of nonlinear stationary solitary wave structures.

So, we want to find a solution of the nonlinear equations (61) and (62) (a non dissipative case $\nu = \sigma_P = 0$) in the form of stationary regular waves $\overline{\psi} = \psi(\eta, z)$ and $R = R(\eta, z)$, propagating along the parallel (along the x-axis) with a constant velocity U = const without changing its form, where $\eta = x - U\tau$. Moreover, we consider the case when the wave structures propagate on the background mean zonal wind, which has the non-uniform velocity.

In the non-dissipative case ($\nu = \sigma_P = 0$), passing to above mentioned auto model variables η and z and considering that in this case $\partial / \partial\tau = -U\partial / \partial\eta$, the system of equations (61), (62) can be written as:

$$-U\frac{\partial}{\partial\eta}\left(\Delta\Psi - \frac{\Psi}{4H^2}\right) + \frac{\partial R}{\partial\eta} + J(\Psi, \Delta\Psi) = 0, \tag{69}$$

$$-U\frac{\partial R}{\partial\eta} - \omega_g^2\frac{\partial\Psi}{\partial\eta} + J(\Psi, R) = 0. \tag{70}$$

Here we have introduced a new feature of the stream function

$$\Psi(\eta, z) = \Phi_0(z) + \overline{\psi}(x, z), \tag{71}$$

and the velocity potential $\Phi_0(z)$ of the background zonal shear flow through the notation:

$$v_0(z) = -\frac{d\Phi_0(z)}{dz}. \tag{72}$$

Providing the so-called vector integration, according to (Aburjania, 2006), the general solution of equation (70) can be presented as:

$$R(\eta, z) = \omega_g^2 z + F(\Psi + Uz), \tag{73}$$

where $F(\xi)$ is the arbitrary function of its argument. Next, substituting (73) into (69) and performing the similar transformation we get a nonlinear equation in the form of the Jacobian:

$$J\left(\Delta\Psi + U\int\frac{dz}{4H^2} + \frac{dF(\Psi + Uz)}{d(\Psi + Uz)}z, \Psi + Uz\right) = 0. \tag{74}$$

The general solution of (74) has the form (Aburjania, 2006):

$$\Delta\Psi + U\int\frac{dz}{4H^2} + \frac{dF(\Psi + Uz)}{d(\Psi + Uz)}z = G(\Psi + Uz), \tag{75}$$

where $G(\xi)$ - a new arbitrary function of its argument.

As it was mentioned earlier, the results of observations and experiments show that vortex streets of various forms can be generated in ordinary liquid and plasma environment in the presence of the shear flow, as a consequence of the nonlinear saturation of Kelvin-Helmholtz instability. Such structures may occur if the asymptotic form of the function $G(\xi)$ in equation (75) is nonlinear (Petviashvili and Pokhotelov, 1992; Aburjania, 2006).

We assume that the nonlinear structure move by a velocity U that satisfies the following condition:

$$U\int\frac{dz}{4H^2} + \frac{dF(\Psi + Uz)}{d(\Psi + Uz)}z = 0. \tag{76}$$

It is obvious that (76) holds for IGW at only case when the function $F(\xi)$ is a linear function of its argument over the plane x, z, i.e. $F = -U(\Psi + Uz)/(4H^2)$. In this case, choosing an arbitrary function G as the following nonlinear function $G(\xi) = \psi_0^0\kappa^2(\exp(-2\xi/\psi_0^0)$ (Petviashvili and Pokhotelov, 1992; Aburjania, 2006), equation (75) reduces to:

$$\Delta(\Psi + Uz) = \psi_0^0\kappa^2\exp[-2(\Psi + Uz)/\psi_0^0]. \tag{77}$$

Now let's choose an expression for the stream function of the background shear flow in the form:

$$\Phi_0(z) = Uz + \psi_0^0\ln(\kappa_0 z). \tag{78}$$

Here ψ_0^0 characterizes the amplitude of the background structure, but $2\pi/\kappa$ и $2\pi/\kappa_0$ presents the characteristic size of the vortex structure and parameter of non-uniform background shear flow, respectively.

Given (71) and using (78), the vorticity equation (77) can be transformed into:

$$\Delta\overline{\psi} = \psi_0^0\kappa_0^2\left[\frac{\kappa^2}{\kappa_0^2}e^{-2\overline{\psi}/\psi_0^0} - 1\right]. \tag{79}$$

This equation has the solution (Mallier and Maslow, 1993):

$$\overline{\psi}(\eta, z) = \psi_0^0\ln\left[\frac{ch(\kappa z) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)}{ch(\kappa_0 z)}\right], \tag{80}$$

which is a street of the oppositely-rotating vortices. Substituting (80) and (78) into expression (71), we finally obtain the solution:

$$\Psi(\eta, z) = Uz + \psi_0^0\ln[ch(\kappa z) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)]. \tag{81}$$

From equations (81), (78) and (59) we obtain the following expressions for the components of the medium velocity and shear flow, respectively:

$$V_x(\eta, z) = -U - \psi_0^0 \kappa \frac{sh(\kappa z)}{ch(\kappa z) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)}, \tag{82}$$

$$V_z(\eta, z) = -\psi_0^0 \kappa \frac{\sqrt{1-\kappa_0^2}\sin(\kappa\eta)}{ch(\kappa z) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)}, \tag{83}$$

$$v_0(z) = -U - \psi_0^0 \kappa_0 th(\kappa_0 z), \tag{84}$$

At $\kappa_0 = 1$ the solution (81) describes the background flow to the type of shear zonal flow (84). At $\kappa_0^2 < 1$ in the middle of the zonal flow (84) the longitudinal vortex street will form (Fig. 3). Solution (82), (83) with closed streamlines in the form of "cat's eyes" was first obtained by Lord Kelvin.

It must be mentioned that the nonlinear stationary equations (74), (75) also have an analytical solution in the form of a Larichev-Reznik type dipole pair of cyclone-anticyclone (Petviashvili and Pokhotelov, 1992; Aburjania, 2006) and vortex transverse chains (Aburjania et al., 2005).

7. Energy transfer by the vortex structures

In the dynamic equations of IGW structures (61) and (62) the source of convergence of external energy due to shear flow (non-uniform wind), the terms with $v_0(y)$, and divergence sources of energy due to dissipative processes in the environment - terms of induction σ_P and viscous ν damping are included obvious. The above mentioned nonlinear solitary vortex structures can self-sustain only at the existence of an appropriate balance between the convergence and divergence of energy in the wave perturbations in the ionosphere medium.

Further, we obtain the energy transport equation for the IGW vortex structures. With this purpose we multiply the equation (61) by $\overline{\Psi}$ and (62) – by ‘R, then integrate them according to x and z. After performing simple transformations of obtained relations we finally get the regularities of the dynamics of energy of the IGW structures:

$$\frac{\partial E}{\partial t} = \int v_0'(z) \frac{\partial \overline{\Psi}}{\partial x}\frac{\partial \overline{\Psi}}{\partial z} dxdz - \frac{\sigma_P B_{0y}^2}{\rho_0} \int \left(\frac{\partial \overline{\Psi}}{\partial x}\right)^2 dxdz - \frac{\sigma_P B_0^2}{\rho_0} \int \left[\left(\frac{\partial \overline{\Psi}}{\partial z}\right)^2 + \frac{\overline{\Psi}^2}{4H^2}\right] dxdz -$$
$$\nu \int \left[\left(\frac{\partial^2 \overline{\Psi}}{\partial x^2}\right) + \left(\frac{\partial^2 \overline{\Psi}}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2 \overline{\Psi}}{\partial x \partial z}\right)^2\right] dxdz, \tag{85}$$

where

$$E = \frac{1}{2}\int \left[\left(\nabla \overline{\Psi}\right)^2 + \frac{\overline{\Psi}^2}{4H^2} + \frac{R^2}{\omega_g^2}\right] dxdz, \tag{86}$$

presents energy of the nonlinear internal-gravity vortex structure.

Let’s mention, that the equation (85) is valid for linear as well as for nonlinear stage of evolution of IGW perturbations. In this equation the first term of the right hand side describes transient sway-generation of the IGW structures due to the shear instability; the second and the third terms – an induction damping of the wave disturbances due to Pedersen conductivity, and the last term describes the viscous damping of the perturbations. According to (85), for generation of the structures it is necessary the velocity of the shear flow to have at least the first derivative with respect to vertical coordinate different from zero ($v_0'(z) \neq 0$).

As noted in section 3.1, the considered IGW perturbations in the linear mode have eigen frequencies (22) and propagate along the Earth's parallel (along the x axis). The induction and viscous damping takes energy from these IGW structures and heat the ionospheric environment with a decrement γ (23), where $k_x \approx 2\pi / L_x$, $k_z \approx 2\pi / L_z$. In this case, shear flow temporarily supply the medium with energy, causing generation - swing of IGW structures and the development of shear instability with a characteristic growth increment γ_A (26).

Thus, the non-uniform zonal wind or shear flow can transiently generate and / or intensify the internal gravity structures in the ionosphere and contribute to self-sustaining of IGW vortices when $\gamma_A \geq \gamma$. According to section 3.1, the condition of nonlinear self-sustaining of IGW vortex structures at the levels of F-region of the ionosphere the condition ($\gamma_A \geq \gamma$) is fulfilled, even with some reserve, and considered vortex structures are long-lived.

Thus, the revealed internal gravity vortices in the ionosphere are sufficiently long-lived, so they can play a significant role in the transport of solid matter, heat, energy and form strong turbulence state in the medium (Aburjania et al., 2009).

8. Discussion and conclusion

In this article the linear stage of generation and further nonlinear evolution of IGW structures in the dissipative stably stratified ($\omega_g^2 > 0$) ionosphere in the presence of shear flow (non-uniform zonal wind) is studied. A model system of dynamic nonlinear equations describing the interaction of internal gravity structures with viscous ionosphere, non-uniform local zonal wind, and the geomagnetic field is obtained. On the basis of analytical solutions and theoretical analysis of the corresponding system of dynamic equations a new mechanisms of linear transient pumping of shear flow energy into that of the wave perturbation, wave amplification (multiple times), self-organization of nonlinear wave perturbations into the solitary vortex structures and the transformation of the perturbation energy into heat is revealed.

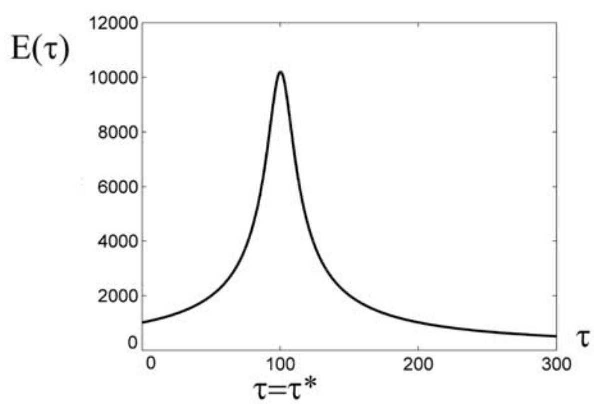

Fig. 1. Evolution of the non-dimensional energy density $E(\tau)$ (formulae (54)) for the initial parameters: $k_0 = 10$, $S = 0.1$.

A necessary condition for shear instability of IGW at their interaction with local non-uniform zonal wind, which is a generalization of the Rayleigh condition, is obtained.

The equation of energy transfer by nonlinear wave structure in the dissipative ionosphere is established. Based on the analysis of this equation it is revealed that the IGW structure effectively interacts with the local background non-uniform zonal wind and self-sustained by the shear flow energy in the ionosphere.

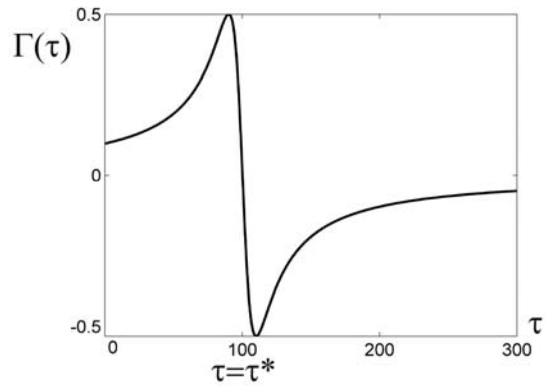

Fig. 2. Increment of shear instability $\Gamma(\tau)$ (formulae (56)), as function of time for the parameters: $k_0 = 10$, $S = 0.1$.

Linear amplification of IGW perturbation is not exponential as in the case of the AGW in the inverse-unstably stratified ($\omega_g < 0$, when IGW can not be generated) atmosphere (Aburjania, 1996)), but in algebraic-power law manner. Intensification of IGW is possible temporarily, for certain values of environmental parameters, shear and waves, which form an unusual way of heating of the shear flow in the ionosphere: the waves draw their energy from the shear flow through a linear drift of SFH in the wave number space (fragmentation of disturbances due to scale) and pump energy into the region of small-scale perturbations, i.e. in the damping region. Finally, the dissipative processes convert this energy into heat. The process is permanent and can lead to strong heating of the medium. Intensity of heating depends on the level of the initial disturbance and the parameters of the shear flow.

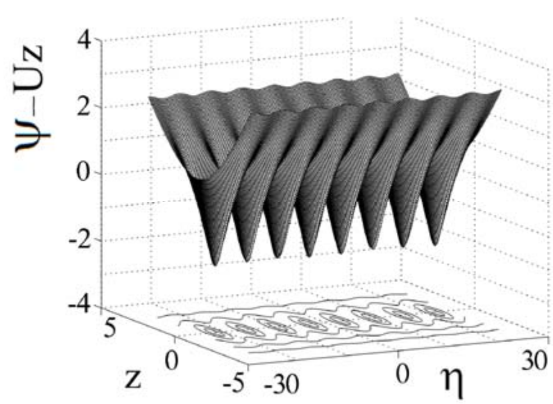

Fig. 3. Relief and current lines in the rest frame of the vortices $\Psi(\eta, y) - Uy$, calculated from formula (81) for $\psi_0^0 = 1$, $k = 1$, $æ_0 = 0.5$ (the longitudinal vortex street).

A remarkable feature of the shear flow is the dependence of the frequency and wave number of perturbations on time $k_z = k_z(0) - k_x S\tau$, $k(\tau) = (k_x^2 + k_z^2(\tau))^{1/2}$. In particular, frequency and wave number transient growth leads to a reduction of scales of the wave disturbances due to time in the linear regime and, accordingly, to energy transfer into a short scale region - the dissipation region. On the other hand a significant change in the frequency range of the generated disturbances stipulates in the environment the formation of a broad range of spectral lines of the perturbations, which is linked to the linear interactions and not to the strong turbulent effects. Moreover, amplification of the SFH perturbation and broadening of wave modes' spectra occur in a limited period of time (transient interval), yet satisfied the relevant conditions of amplification and a strong enough interaction between the modes.

It should be emphasized that the detection of the mechanism of the intensification and broadening of the spectrum of perturbations became possible within the non-modal mathematical analysis (these processes are overlooked by more traditional modal approach). Thus, non-modal approach, taking into account the nonorthogonality of the eigenfunctions of the linear wave dynamics, proved to be more appropriate mathematical language to study the linear stage of the wave processes in shear flows.

The frequency of considered linear IGW perturbations varies in the interval of $10^{-4}\,c^{-1} < \omega_0 < 1.7\times 10^{-2}\,c^{-1}$ and includes low-frequency range of AGW. Wavelength lies in the interval $\lambda \sim 100\text{м} \div 10\text{km}$, the period – from 5 minutes to - 3 hours. Considering intermediate values of the IGW wavelengths ($k \sim 1/H,\ H \sim 10$ km; $\omega \sim \omega_g \sim 10^{-2}s^{-1}$) we find that the group and phase velocities are of the same order $V_g \sim V_p \sim \omega_g H \sim 10^{-2}\,s^{-1} \times 10^4\,m \sim 10^2\,m/s$. This estimation agrees with existing observations and they move with velocity ($0.1 \div 200$) m/s in a random direction along the horizontal lines, depending on daytime and nighttime conditions. IGW is characterized by an exponential growth of the amplitude of the perturbed velocity at the vertical propagation in an environment with exponentially decaying vertical equilibrium density and pressure (Hines, 1960; Gossard, Hook, 1978). According to observational data, IGW disturbances manifest themselves in a wide range of heights - from the troposphere to the upper ionosphere heights $z \le 600$ km (Gossard and Hook, 1975; Francis, 1975; Rishbet and Fukao, 1995; Hecht et al., 2010). At ionospheric altitudes (above 90 km) the conductive medium strongly impacts on the IGW, causing its remarkable damping due to local Pedersen currents.

On the basis of analytical solutions of nonlinear dynamical equations it's shown that the internal-gravity waves organize themselves (due to the shear flow energy) in the form of stationary solitary vortex structures. The solution of the nonlinear equations has an exponential asymptotic behavior $\sim \exp(-\kappa|r|)$ at $|r| \to \infty$, i.e. structures are strongly localized along the plane transverse to the Earth's surface. Depending on the type of velocity profile of the zonal shear flow (wind) $v_0(z)$, the generated nonlinear structures maybe the monopole solitons, cyclone, anticyclone, dipole cyclone-anticyclone pair, longitudinal vortex streets or transverse vortex chain in the background of non-uniform zonal wind (see also Aburjania, et al., 2005). The presence of spatially non-uniform winds in the ionosphere gives IGW the properties of self-organization and self-sustaining in the form of the aforementioned nonlinear solitary vortex structures of different shapes.

Phase velocity of linear IGW occupies a range: $-V_{max} \le V_p \le V_{max}$ in an incompressible atmosphere, where $V_{max} = 2H\omega_g = 2(gH)^{1/2}$. This means that if the source (for example, the above mentioned nonlinear vortex structure) moves along x axes at a velocity greater than V_{max}, the source does not come in resonance with the linear internal gravity waves. Nonlinear vortices, moving faster than the corresponding linear waves, can retain their nonlinear amplitude as far as they do not lose energy by radiation of linear waves. It means that these sources can not excite a linear wave due to

Cherenkov mechanism and can retain its initial energy (Stepanyants and Fabrikant, 1992). Thus, these vortex structures can be generated, self-sustained and propagated with velocity $|U| > V_{max}$ along the horizon in any direction. For the height of the atmosphere $H \approx 4.5 \div 6$ km, we can estimate the value of maximal speed of linear IGW - $V_{max} \approx 440$ m / sec. Thus, the identified vortex structures are supersonic and do not loose energy by radiation of linear IGW in the velocity $|U| < V_{max}$ m / s region.

It should be noted that the discussed nonlinear two-dimensional vortex structures are very different from the atmospheric Rossby-type vortices (Larichev and Resnick, 1976; Aburjania, 2006). The main difference is that the motion velocity of our vortices is completely symmetric, i.e. the structures can move with velocities greater than the maximum phase velocity of linear waves in any horizontal direction. While Rossby vortices can move to the west only at the velocities exceeding the maximum velocity of Rossby waves. In the East such vortices can move with any speed as far as the linear Rossby waves do not propagate in this direction. In addition, we assumed that the atmospheric-ionospheric medium is isothermal. In case, when the equilibrium temperature T_0 of the medium is not constant, in the expression of maximum velocity of linear IGW $V_{max} = 2c_s(\gamma-1)^{1/2}/\gamma$, the term $\gamma - 1$ must be replaced by $\gamma - 1 + H(dT_0/dz)/T_0$. Then, for temperatures, coinciding instability threshold (i.e. $d \ln T_0 / dz < 0$) (Stenflo and Stepaniants,1995), and for $\gamma = 1.4$ we get $V_{max} < |U| << c_s \approx 330$ m/s a nonlinear stationary IGW structure can be generated in atmospheric-ionospheric media .

Nonlinear vortex structures of large amplitude, very similar to those theoretically identified by us, were found at atmosphere-ionosphere layers with satellite and ground observations and analyzed in the papers (Bengtsson and Lighthill, 1985; Ramamurthy et al., 1990; Cmyrev et al., 1991; Nezlin, 1994; Shaefer et al., 1999). The motion of medium particles trapped by vortex structures is characterized by a non-zero vorticity $\nabla \times \mathbf{V} \neq 0$, i.e. particles rotate along the closed trajectories in the nonlinear structures. Characteristic value of rotational velocity U_c is of the order or greater than the structure velocity as a whole U, $U_c \geq U$. In this case, the structures trap the medium particles (whose number is comparable to the number of passing particles) and moving in the environment, transfer these rotating trapped particles. Therefore, being long-lived entities, IGW vortex structures can play a significant role in the process of transfer of mass, heat, energy and in the creation of macro turbulent state of ionosphere (Aburjania et al., 2009). In particular, the vortex structure can play the role of "turbulent agent" or elements of horizontal macroscopic turbulent exchange processes in general circulation of the ionosphere. Coefficient of horizontal turbulent eddy exchange can be estimated using the Obukhov-Richardson formula (Monin and Yaglom, 1967): $K_T \approx 10^{-6} d^{4/3}$ m^2/s. So, for the typical size of vortices $d \sim 10$ km, we find that $K_T \approx 10^2$ m^2/s. This estimate (which must be regarded as an upper limit) shows that the exchange processes between the upper and lower latitudes, the meridional heat transport from north to south in the ionosphere can have macro-turbulent vortex nature (note that in the ionosphere, the polar region is warmer than equatorial).

IGW structures are eigen degrees of freedom of the ionospheric resonator. Therefore, influence of external sources on the ionosphere above or below (magnetic storms, earthquakes, artificial explosions, etc.) will excite these modes (or intensified) in the first, (Aburjania and Machabeli, 1998). For a certain type of pulsed energy source the nonlinear solitary vortical structures will be generated (Aburdjania, 1996; Aburdjania, 2006), which is confirmed by experimental observations (Ramamurthy et al., 1990; Cmyrev et al., 1991; Nezlin, 1994; Shaefer et al., 1999; Sundkvist et al., 2005). Thus, these wave structures can also be the ionospheric response to natural and artificial activity.

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Генерация, интенсификация и само-организация внутренних-гравитационных волновых структур в земной ионосфере с направленным сдвиговым ветром

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Резюме

Изучена генерация, интенсификация и дальнейшая нелинейная динамика внутренних гравитационных волн (ВГВ) в устойчиво-стратифицированной диссипативной ионосфере с неоднородным зональным ветром (сдвиговым течением). В сдвиговых течениях операторы линейных задач являются несамосопряженными, а соответствующие собственные функции – неортогональными, поэтому канонически-модальный подход мало пригоден при изучении таких движений. Более адекватным для таких задач становится немодальный математический анализ. На основе немодального математического анализа получены уравнения динамики и переноса энергии ВГВ возмущений в ионосфере со сдвиговым течением. Выводится необходимое условие критерий неустойчивости сдвигового течения в ионосферной среде. Найдено точное аналитическое решение как линейных, так и нелинейных динамических уравнений рассматриваемых задач. Найден инкремент сдвиговой неустойчивости ВГВ. Выявлено, что временное усиление ВГВ возмущений происходит не экспоненциально, а алгебраическим – степенным образом. Частота и волновой вектор генерируемых ВГВ мод являются функциями времени. Так что, в ионосфере со сдвиговым течением, волновые возмущения с широким спектром порождаются с линейным эффектом даже тогда, когда отсутствуют нелинейные и турбулентные эффекты. Проанализирована эффективность линейного механизма усиления ВГВ при их взаимодействии с неоднородным зональным ветром. Показано, что ВГВ эффективно черпают энергию сдвигового течения в линейной стадии эволюции и существенно увеличивают (на порядок) свою энергию и амплитуду. С увеличением амплитуды включается нелинейный механизм самолокализации, и процесс заканчивается сам-организацией нелинейных, сильно локализованных ВГВ вихревых структур. Тем самим появляется новая степень свободы системы и путь эволюции возмущений в среде со сдвиговым течением. В зависимости от вида профиля скорости сдвигового течения нелинейные ВГВ структуры могут

быть или чисто монопольным вихрем, или вихревой дорожкой, или вихревой цепочкой на фоне неоднородного зонального ветра. Накопление таких вихрей в ионосферной среде может создавать сильнотурбулентное состояние.

შიდა-გრავიტაციული ტალღური სტრუქტურების გენერაცია, ინტენსიფიკაცია და თვით-ორგანიზაცია დედამიწის იონოსფეროში არაერთგვაროვან ქარებთან ურთიერთქმედებისას

გ. აბურჯანია, ხ. ჩარგაზია

რეზიუმე

შესწავლილია შიდა-გრავიტაციული ტალღების (შგტ) გენერაცია, ინტენსიფიკაცია და შემდგომი არაწრფივი დინამიკა მდგრადად სტრატიფიცირებულ დისიპაციურ იონოსფეროში, რომელზეც ზემოქმედებს არაერთგვაროვანი ზონალური ქარი (წანაცვლებითი დინება). წანაცვლებითი დინებების წრფივი დინამიკის აღმწერ წრფივ განტოლებებში შემავალი ოპერატორები არ არიან თვით-შეუღლებულნი და შესაბამისი საკუთარი ფუნქციები არიან არაორთოგონალურნი. ამიტომ, ასეთი მოძრაობის აღსაწერად კანონიკური, მოდალური მიდგომა ნაკლებად გამოდგება. მზგავსი ამოცანების ამოსახსნელად უფრო ადექვატური გამოდგა ე.წ. არამოდალური მათემატიკური ანალიზი. არამოდალური მიდგომის საფუძველზე მიღებულია შგტ შეშფოთებების დინამიკის აღმწერი და ენერგიის გატანის განტოლებები არაერთგვაროვანი ქარებით მართულ იონოსფეროში. მიღებულია წანაცვლებითი დინებების იონოსფეროში არამდგრადობის აუცილებელი პირობა-კრიტერიუმი. განსაზღვრულია წანაცვლებითი არამდგრადობის ინკრემენტი. აგებულია ამოცანის აღმწერი წრფივი და არაწრფივი დინამიკურ განტოლებათა სისტემის ზუსტი ანალიზური ამონახსნები. გამოვლენილია, რომ შგტ შეშფოთებების გაძლიერება დროის მიხედვით ხდება არა ექსპონენციალურად, არამედ ალგებრულად-ხარისხობრივად. გენერირებული მოდების სიხშირე და ტალღური ვექტორი ხდებიან დროის ფუნქცია. ასე, რომ წანაცვლებითი დინებებისას იონოსფეროში წრფივი პროცესებით ეფექტურად აღიძვრებიან ფართო დიაპაზონის ტალღური შეშფოთებები მაშინაც კი, როცა არაწრფივი და ტურბულენტური პროცესები არ წარმოიშობიან. გაანალიზებულია შგტ გაძლიერების ეფექტურობა მათი არაერთგვაროვან ზონალურ ქარებთან ურთიერთქმედებისას. ნაჩვენებია, რომ საწყის წრფივ სტადიაზე შგტ დროებით მაგრამ ეფექტურად ართმევს ენერგიას წანაცვლებით დინებას და შესამჩნევად ზრდის (რიგით) თავის ამპლიტუდას და ენერგიას. ამპლიტუდის ზრდისას თამაშში ერთვება არაწრფივი თვითლოკალიზაციის ეფექტი და პროცესი მთავრდება არაწრფივი, ძლიერად ლოკალიზებული შგტ გრიგალური სტრუქტურების თვითორგანიზაციით. მაშასადამე, წანაცვლებითი დინებები სისტემას ანიჭებს ახალ თავისუფლების ხარისხს და შესაბამისად, აჩენს შეშფოთებების ევოლუციის ახალ გზას. წანაცვლებითი დინებების სიჩქარის პროფილისაგან დამოკიდებულებით შგტ სტრუქტურები შეიძლება იყონ ან სუფთა მონოპოლური გრიგალები, ან განივი გრიგალური ჯაჭვები ან და გასწვრივი გრიგალური ბილიკები არაერთგვაროვანი

ზონალური ქარის ფონზე. ასეთი გრიგალური სტრუქტურების იონოსფეროში დაგროვებამ შეიძლება წარმოქმნას ძლიერად ტურბულენტური მდგომარეობა.