Modeling of the frequency spectrum of magnetogradient waves in the equatorial magnetopause in the case of variable electric conductivity of the solar wind plasma

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Abstract

It is known that after a bow shock in front of the day-side of the magnetosphere, i.e. in the magnetosheath, the solar wind flow decelerates. Therefore, the solar wind an anomalous resistance develops, which causes an increase in the magnetic viscosity of the plasma. This effect is especially felt in the focal area of the magnetosheath (the stagnation zone before the magnetosphere) where, according to our discussion, unlike in the peripheral areas of the magnetosheath, the approximation of single-fluid magnetic hydrodynamics (MHD) is not correct. This is especially characteristic of the base of the stagnation zone, which is the central area of the Earth's magnetic boundary layer (the magnetopause). Thus, it will be correct to describe the large scale motion of the solar wind plasma, having finite electric conductivity, with an equation system of double-fluid magnetic hydrodynamics. Generally, working out self-consistent analytical solutions to the magnetic and velocity fields, except in extremely simple cases, is impossible due to mathematical complications. However, there is a solution to this problem in the case of the flow around of the magnetosphere as due to violent deceleration of the solar wind. In the focal area of the magnetosheath it is possible to determine the flow topology in the kinematic approximation. Such a solution enables to solve the equation of the magnetic induction, constituent of the MHD equations system and corresponding to the magnetopause, by analytical approximation methods. Among different kinematic models, which include the solar wind deceleration effect near the critical point of the magnetosphere, Parker's plane (two dimensional) kinematic model for incompressible medium is especially simple [1]. This model and its spatial modification were effectively used in different tasks [2]. Namely, it appeared convenient for obtaining the magnetosphere parameters in quasi-stationary approximation by means of different models of variations in time of the magnetic viscosity of the solar wind [3,4].

Key words: Solar wind, magnetosphere, magnetosheath, critical point, stagnation zone.

Magnetogradient waves of Rossby type. Variation of the thermodynamic parameters of the plasma due to the solar wind deceleration in the magnetosheath may cause different kinematic and hydrodynamic phenomena. Namely, near the dayside boundary of the magnetosphere there is a possibility of generation of so-called magneto- gradient (MG) waves of Rossby's atmospheric planetary wave type. The existence of such waves in the Earth's ionosphere was independently supposed in the papers [5,6]. For activation of the physical mechanism for generating the MG waves it is necessary for the plasma to move with low hydrodynamic velocity or stagnate at the background of transverse electric conductivity (so-called Hall effect) towards the inhomogeneous magnetic field. It appeared that besides in the ionosphere the generation of the MG waves is also possible in the stagnation zone before the magnetosphere where the solar wind velocity becomes commensurable to the electromagnetic drift velocity of the electric component of the plasma [7, 8- Aburdjania, et.al, 2007,9]. In these papers the discrete spectral time characteristics of the MG wave frequencies were obtained as the stationary MHD conditions were considered. Then these

conditions were generalized for a quasi-stationary case. Thus we suppose that it is useful to use the results of the paper [3]. In such case, as it will be shown below, it will be possible to determine continuous spectral time characteristics of the MG waves by means of the quasi-stationary parameters of the magnetopause.

Let us enter the rectangular coordinate system with its zero point in the critical (front) point of the magnetosphere. The **x-** axis is directed to the sun, the **y** -axis coincides with the direction of the geomagnetic field boundary force line perpendicular to the equatorial plane, the **z** -axis is directed to the central equatorial section of the magnetosphere alongside the magnetosphere boundary. It is known that in a double-fluid MHD approximation two types (fast and slow) of MG waves may generate. Such waves may also generate in the stagnation zone of the magnetosheath, especially in the central area of the equatorial magnetopause. The phrasal velocities of these waves are determined by the expressions [8,9]:

A fast MG wave:

$$C_\varphi = C_+ = -\left(\frac{|C_H|}{2} + \sqrt{\frac{C_H^2}{4} + |C_H||C_p'|} \right), \qquad (1)$$

A slow MG wave:

$$C_\varphi = C_- = -\frac{|C_H|}{2} + \sqrt{\frac{C_H^2}{4} + |C_H||C_p'|} \; . \qquad (2)$$

where $C_H = c/(4\pi e n)(\partial H_y/\partial x)$ is a magneto-gradient wave (so-called Khantadze wave), $C_p' = -\beta_H/k_x^2 > 0$ is the Rossby's type wave, $\beta_H = e/(Mc)(\partial H_y/\partial x)$ is the parameter of the magnetic field inhomogeneity (so-called magnetic parameter of Rossby), $k_z = 2\pi/\lambda$ is the wave number connected with the scale of the linear inhomogeneity of the task, e -is the elementary charge, n – the electron density, M – the proton mass, c – the light velocity . In a single-fluid approximation, i.e. when there is no Hall electric conductivity effect, there will be only stable MG wave.

According to (1) and (2) expressions, in order to determine the spectral characteristics of the wave in the equatorial magnetopause by means of the dispersive relationship it is necessary to analytically determine the magnetopause thickness and the distribution of the geomagnetic field in it. In addition, in the first approximation we may use the magnetopause thickness as the inhomogeneity linear scale determining the wave number [8]. Let us refer to the papers [3,4], in which by use of the Parker kinematic model, by analytical method of the Schwec successive approximation [10] the quasi-stationary solutions of the equation of the magnetic field induction are obtained. These solutions correspond to different models of time variation of the electric conductivity of the solar wind. In the previous results the impulsive time variation of either the electric resistance of the solar wind plasma, or the parameter depended on it - the magnetic viscosity, were not considered [11]. But obtained experimental data proved the possibility of anomalous increase of the electric resistance of the solar wind that has been used in modern computer experiments [12]. Therefore, it is obvious that qualitative and quantitative corrections of the previous results of modeling of magnetopause carried out earlier are necessary.

Let us not take into account the curvilinearity of the extreme force line of the geomagnetic field on the boundary of dayside magnetosphere. In case of such admission for determining topologic image of nonstationary distribution of the magnetic field in the Zhigulev second category plane boundary layer we may use a single-component equation of magnetic induction H

$$\frac{\partial H_y}{\partial t} + u\frac{\partial H_y}{\partial x} + v\frac{\partial H_y}{\partial z} = \lambda_m \frac{\partial^2 H_y}{\partial^2 x} \; . \qquad (3)$$

This equation involves magnetic viscosity λ_m as a coefficient that is defined by σ specific electric conductivity

$$\lambda_m = \frac{c^2}{4\pi\sigma},$$
(4)

Let us use the following expressions for modeling of the impulsive time variation of σ parameter during perturbation of the solar wind

$$1)\ \lambda_m = \lambda_{0m}e^{-\frac{t}{\tau_0}};\qquad 2)\ \lambda_m = \lambda_{0m}(1-e^{-\frac{t}{\tau_0}}),$$
(5)

where λ_{0m} is the value characterizing the magnetic viscosity, τ_0 - the time characterizing the impulsive variation of this parameter. It is obvious that these models are physically similar and show the change of the electric conductivity of the plasma from the finite to the ideal and vice versa.

According to the Shwec successive approximation analytical method suppose that the value of the Earth's dipole magnetic field in the lower boundary of the magnetopause is constant and gradually decreases in latitudinal direction of the δ_H thickness of the magnetic boundary layer. Thus, the infinite upper boundary of the integration may be replaced by the finite thickness of the magnetic boundary layer. Then this parameter is defined in analytically clear form. Such a possibility is main advantage of the Schwec successive approximation.

Thus, we have the following boundary conditions for the (3) equation

$$H_y = H_0, \text{ when } x = 0; \quad H_y = 0, \quad \text{when } x = \delta_H.$$
(6)

Near the critical point of the magnetosphere the velocity field of the noncompressible plasma is determined by the Parker's plane kinematic model

$$u = -\alpha x, \qquad v = \alpha z,$$
(7)

where α is the reverse value of the time characteristic for the overflow of the magnetosphere day side. Further we will use value $\alpha = 0.01$, which corresponds to the velocity of the electromagnetic drift in the stagnation zone V=5 km/ sec in the case of minimal linear scale of this structure: 1000 km. Thus, by means of (7), in case of the (5) model, we will have the equation

$$\frac{\partial H_y}{\partial t} - \alpha x \frac{\partial H_y}{\partial x} = \lambda_m \frac{\partial^2 H_y}{\partial x^2}.$$
(8)

In the (6) boundary conditions, for solving the (8) equation by Shwec method, also the corresponding equation of the (5.2) model and for gaining information on the determination scheme of the magnetopause thickness we may refer to the works [2008]. Therefore, it is quite sufficient to present quasi-stationary expressions of the distribution of the magnetic field over the meridional magnetopause and the boundary layer thickness ($'$ means the time derivative)

1) $\lambda_m = \lambda_{0m}e^{-\frac{t}{\tau_0}}$

$$\frac{H_y}{H_0} = \left(1 - \frac{x}{\delta_H}\right) + \lambda_{0m}^{-1}e^{\frac{t}{\tau_0}}\left[\left(\frac{\delta_H'}{\delta_H^2}\frac{x^3}{6} - \frac{\delta_H'}{6}x\right) + \alpha\left(\frac{x^3}{6\delta_H} - \frac{\delta_H}{6}x\right)\right]$$
(9)

$$\delta_H = (3\lambda_{0m}\alpha^{-1})^{1/2}\left[e^{-\alpha t} + \left(1 - \frac{1}{\alpha\tau_0}\right)^{-1}\left(e^{-\frac{t}{\tau_0}} - e^{-\alpha t}\right)\right]^{1/2},$$
(10)

2) $\lambda_m = \lambda_{0m}(1-e^{-\frac{t}{\tau_0}})$

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$$\frac{H_y}{H_0} = (1 - \frac{x}{\delta_H}) + \lambda_{0m}^{-1}\left[(1 - e^{-\frac{t}{\tau_0}})\right]^{-1}\left[(\frac{\delta_H'}{\delta_H^2}\frac{x^3}{6} - \frac{\delta_H'}{6}x) + \alpha(\frac{x^3}{6\delta_H} - \frac{\delta_H}{6}x)\right],$$ (11)

$$\delta_H = (3\lambda_{0m}\alpha^{-1})^{1/2}\left[(1 - e^{-\alpha t}) + \left(1 - \frac{1}{\alpha\tau_0}\right)^{-1}\left(e^{-\frac{t}{\tau_0}} - e^{-\alpha t}\right)\right]^{1/2}.$$ (12)

The existence of the magnetopause is provided by the global surface DCF current, which screens the geomagnetic field from the solar wind. In this way the correctness in regard to the general image of the flow around of the magnetosphere, according to which the magnetopause is the magnetic boundary layer, will be proved. For model 1) we have $\delta_H = (3\lambda_{0m}\alpha^{-1})^{1/2}$ when $t = 0$, according to (10). So, Earth's magnetic boundary layer a priori has a certain thickness in this case, as opposed to case 2) for which $\delta_H = 0$ when $t = 0$. It means that we may use physical analogy at the hydrodynamic boundary layer, inside of which for assessment of the energy changes there are two effective parameters: the thickness of the boundary layer and the thickness of loss of the mechanical impulse. For the MHD boundary layer, as the analogy of these parameters, two characteristics were used: 1) δ_a - the thickness of displacement of magnetic field induction; 2) δ_b - the thickness of magnetic energy displacement [4,11]. According to the explanation the thickness of displacement of magnetic field induction shows the thickness of the induction flow loss by means of comparing the distribution of the magnetic field to the corresponding distribution of the ideal profile in the latitudinal section of the magnetopause. In addition, the thickness of the energy loss of the magnetic field shows the thickness of the lost energy layer by comparing it to the ideal distribution. Generally, these parameters are defined by the following expressions

$$\delta_a = \int_0^{\infty\delta_H}\left(1 - \frac{H}{H_0}\right)dx,$$ (13)

$$\delta_b = \int_0^{\infty\delta_H}\left(1 - \frac{H^2}{H_0^2}\right)dx,$$ (14)

Therefore, it is logical to consider the one of the thicknesses (10), (12), (13) and (14) as the linear scale necessary for quantitative assessment of the MG wave parameters. It is obvious that in (9) and (11) expressions the role of the second approximation is much less compared to the first one. It is obvious that parameters C_H and C_p' will change according to the time as well as transversally to the magnetopause. Consequently, for assessment of the magneto-gradient wave velocity and the Rossby magnetic parameter it is sufficient to consider that the distribution of the magnetic field in the magnetopause is approximated by the linear terms of the expression (9) and (11). Thus, the phase velocities will be defined by the first approximation of Schwec. It means that we may use the following simple expressions for the magneto-gradient wave and the Rossby wave constituent of the expressions (1) and (2)

$$|C_H| = \frac{c}{4\pi e n} \cdot \frac{H_0}{\delta_H}, \qquad C_p' = \frac{eH_0}{Mc\delta_H k_z^2}.$$ (15)

These formulas included δ_H as the linear scale and H_0 as a characteristic value of the magnetic field in the magnetopause. Thus, we can have characteristic values of the phase velocities of in the quasi-stationary magnetopause, the thickness of which is determined by the variation in time of the magnetic viscosity of the plasma. Let us use the following parameters of an average disturbed solar wind plasma near the magnetosphere boundary: $\lambda_m = 10^{14}$cm^{2}s^{-1} and $n = 20$particle.cm^{-3}. The characteristic value $H_0 = 2 \cdot 10^{-4}$ gauss corresponds to the unperturbed value of the geomagnetic dipole at the low boundary of the magnetopause. Now, we can determine

the value of the wave number by means of the minimal linear scale of the stagnation zone boundary $-l \approx 10^{9}$ cm, i.e. $k_y = 2\pi \cdot 10^{-9} cm$ [8]. Let us consider the characteristic time of the magnetospheric substorm development – 500s as the characteristic scale of the τ_0 time. By means of these parameters the following characteristic values are defined: $C_H \approx 5 \cdot 10^{6} cm.s^{-1}$ and $C_p^i \approx 2 \cdot 10^{6} cm.s^{-1}$. Therefore it becomes possible to define the velocities of the fast and slow magneto-gradient waves, after which we recieve the frequency spectrum characteristic of the magneto-gradient waves by means of expressions: $\omega_+ = k_y C_+$ and $\omega_- = k_y C_-$. As a matter of fact, there is not a great difference between these values, due to which in the equatorial magnetopause an overlap of the frequency spectrum of the fast and slow magneto-gradient waves will take place
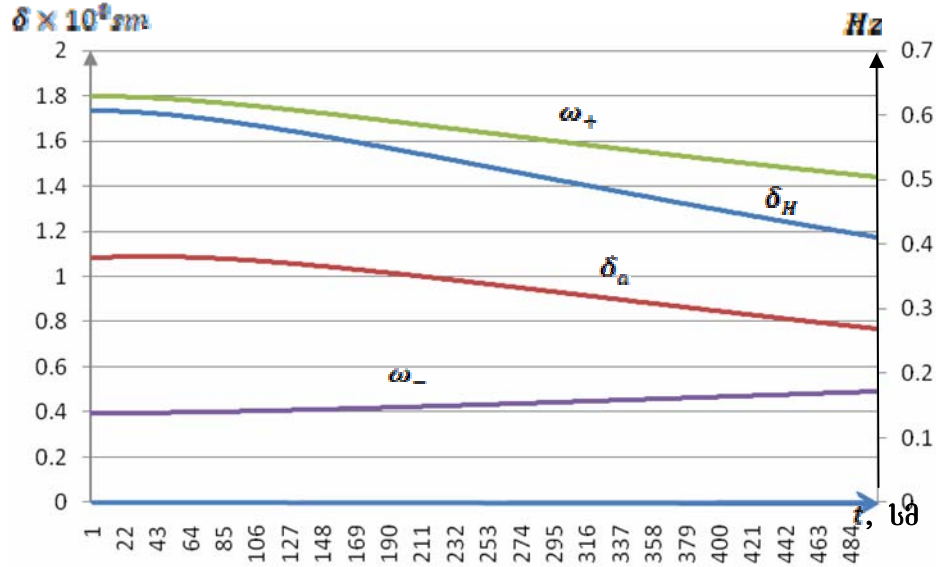
1) $\lambda_m = \lambda_{0m} e^{-\frac{t}{\tau_0}}$;

Fig.1

2) $\lambda_m = \lambda_{0m}(1 - e^{-\frac{t}{\tau_0}})$
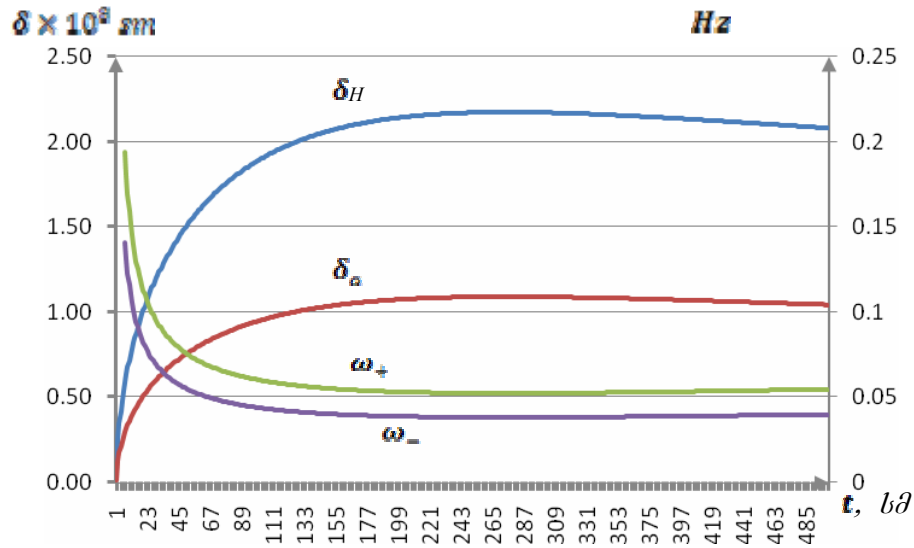

Fig.2

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The figures 1-2 show the scheme of variation of the ω_+ parameter and the schemes of the δ_H and δ_σ parameters during the characteristic time interval for both models of magnetic viscosity. Thus, in case of $\lambda_m = \lambda_{0m}e^{-t/\tau_0}$ the characteristic value of ω_+ is almost constant- 0.5Hz. However, when $\lambda_m = \lambda_{0m}(1 - e^{-t/\tau_0})$ we have the characteristic spectrum value of the ω_+ (0.2-0.05)Hz. For comparison we may use value $\omega_+ \approx 5 \cdot 10^{-3}$ Hz, which was obtained in the approximation of the minimal size stagnation zone in the paper [8]. It is obvious that in case we use the δ_σ as a characteristic linear scale instead of the δ_H there will not be much difference in quantitative results.

Thus, in the limits of the above presented model in the equatorial magnetopause a significant change of the frequency spectrum of the magneto-gradient waves characteristic of the stagnation zone may occur. It may be caused by the change in the thickness of the magnetopause and the specification of the distribution of the magnetic field in it due to variation in time of the magnetic viscosity of the solar wind. It is noteworthy that in such a case the frequency spectrum characteristic of the stagnation zone contains the frequencies of short and middle period geomagnetic pulsations.

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მაგნიტოგრადიენტული ტალღების სიხშირეთა სპექტრის მოდელირება ეკვატორიალურ მაგნიტოპაუზაზე მზის ქარის პლაზმის ცვლადი ელექტრული გამტარებლობის შემთხვევაში

მ. ჩხიტუნიძე,   ზ. კერესელიძე

რეზიუმე

ცნობილია, რომ გარდამავალ არეში მაგნიტოსფეროს დღის მხარის წინ დარტყმითი ტალღის შემდეგ მზის ქარის დინება შენელებულია. შესაბამისად, მზის ქარში ვითარდება ანომალური ელექტრული წინაღობა, რაც პლაზმის მაგნიტური სიბლანტის ზრდას იწვევს. ეს ეფექტი განსაკუთრებით საგრძნობია გარდამავალი არის ფოკალურ ნაწილში (მაგნიტოსფეროს წინამდებარე სტაგნაციის ზონა), სადაც, განსხვავებით პერიფერიული არეებისაგან, ჩვენი შეხედულებით სამართლიანი აღარ არის ერთსითხოვანი მაგნიტური ჰიდროდინამიკის (მჰდ) მიახლოება. ეს განსაკუთრებით ეხება სტაგნაციის ზონის ფუძეს, ანუ დღედამიწ ის მაგნიტური სასაზღვრო ფენის (მაგნიტოპაუზა) ცენტრალურ ნაწილს. ამიტომ აქ კორექტულია სასრული ელექტრული გამტარებლობის მქონე   მზის ქარის პლაზმის   მსხვილმასშტაბოვანი   მოძრაობის   აღწერა   ორსითხოვანი   მაგნიტური ჰიდროდინამიკის განტოლებათა სისტემით. საზოგადოდ, მაგნიტური ველისა და სიხჩარეთა ველის თვითშეთანხმებული ანალიზური ამონახსნის მიღება, განსაკუთრებით მარტივი შემთხვევების გარდა, შეუძლებელია მათემატიკური გართულებების გამო. თუმცა, მაგნიტოსფეროს გარსდენის შემთხვევაში, მზის ქარის მკვეთრი დამუხრუჭების გამო, არსებობს გამოსავალი: გარდამავალი არის ფოკალურ ნაწილში შესაძლებელია დინების ტოპოლოგიის განსახღვრა კინემატიკურ მიახლოებაში. ასეთი ხერხი შესაძლებელს ხდის მაგნიტოპაუზ ის შესაბამისი, მჰდ განტოლებათა სისტემის შემადგენელი, მაგნიტური ინდუქციის განტოლების ამოხსნას მიახლოებითი ანალიზური მეთოდით. მაგნიტოსფეროს კრიტიკული წერტილის მახლობლად მზის ქარის დამუხრუჭების პროცესის აღმწერ სხვადასხვა კინემატიკურ მოდელებს შორის განსაკუთრებული სიმარტივით გამოირჩევა პარკერის ბრტყელი (ორ განზომილებიანი) კინემატიკური მოდელი უკუმშვადი გარემოსათვის. მოცემულ ნაშრომში ეს მოდელი გამოყენებულია ეკვატორიალურ მაგნიტოპაუზაზე როსბის პლანეტერული ატმოსფერული ტალღების ტიპის მაგნიტოგრადიენტული ტალღების მახასიათებელი პარამეტრების მისადებად. ამისათვის, გამოყენებული იყო მზის ქარის მაგნიტური სიბლანტის დროში იმპულსური ცვალიდების მოდელები და შვეცის მიმდევრობითი მიახლოებების ანალიზური მეთოდი. განისაზღვრა მაგნიტოგრადიენტული ტალღების ფაზური სიჩქარის მახასიათებელი სიდიდე და მოხდა მათი სიხშირეთა უწყვეტი (კვაზისტაციონარული) სპექტრის მოდელირება, რომელიც მოიცავს მოკლე და საშუალო პერიოდიანი რეგულარული გეომაგნიტური პულსაციების დიაპაზონს.

Моделирование спектра частот магнитоградиентных волн на экваториальной магнитопаузе в случае переменной электропроводности плазмы солнечного ветра

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Резюме

Известно, что в переходном слое, на дневной стороне магнитосферы, после прохождения фронта ударной волны, происходит торможение течения солнечного ветра. Соответственно, в солнечном ветре развивается аномальное электрическое сопротивление, что вызывает увеличение его магнитной вязкости. Этот эффект находится в фокальной части переходного слоя (застойная зона перед магнитосферой), где в отличие от периферийных областей, по нашему мнению, одножидкостное приближение магнитной гидродинамики (МГД) является несправедливым. Такое замечание особенно относится к основанию застойной зоны т.е. к центральной части (магнитопаузы) магнитного пограничного слоя Земли. Поэтому, тут для описания крупномасштабного движения плазмы солнечного ветра, имеющей конечную электрическую проводимость, пользуются уравнениями двухжидкостной магнитной гидродинамики. Вообще, получение самосогласованного аналитического решения магнитного поля и поля скоростей, кроме особенно простых случаев, является невозможным из-за математических осложнений. Однако, в случае обтекания магнитосферы, из-за резкого торможения солнечного ветра, существует выход - топологию течения в фокальной части переходного слоя можно определить в кинематическом приближении. Такой способ позволяет одну из составляющих системы МГД уравнений магнитопаузы (в частности, уравнение магнитной индукции) найти преб緣женным методом. Среди различных кинематических моделей, которые описывают процесс торможения солнечного ветра вблизи критической точки магнитосферы, особой простотой выделяется плоская (двухмерная) кинематическая модель Паркера для несжимаемой среды. В данной работе, эта модель используется для определения характерных параметров магнитоградиентных волн Россби на экваториальной магнитопаузе. Для этого были использованы модели импульсного изменения магнитной вязкости солнечного ветра и аналитический метод последовательных приближений Швеца. Были определены характерные величины фазовой скорости магнитоградиентных волн, при помощи которых было проведено моделирование непрерывного (квазистационарного) спектра частот, который включает диапазон регулярных средне и коротка периодных геомагнитных пульсаций.