Journal of the Georgian Geophysical Society,
Issue B. Physics of Atmosphere, Ocean, and Space Plasma, v.16b, 2013, pp. 53-75

Generation of zonal flow and magnetic field by coupled Rossby – Alfvén – Khantadze waves in the Earth's ionospheric E – layer

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Abstract

It is shown that in the Earth's weakly ionized ionospheric E-layer with the dominant Hall conductivity new type of coupled Rossby – Alfvén – Khantadze (CRAK) electromagnetic (EM) planetary waves attributable by latitudinal inhomogeneity of both the Earth's Coriolis parameter and the geomagnetic field can exist. Under such coupling new type of dispersive Alfvén waves is revealed. Generation of sheared zonal flow and magnetic field by CRAK EM planetary waves is investigated. The nonlinear mechanism of the instability is based on the parametric excitation of zonal flow by interacting four waves leading to the inverse energy cascade in the direction of longer wavelength. A 3D set of coupled equations describing the nonlinear interaction of pumping CRAK waves and zonal flow is derived. The growth rate of the corresponding instability and the conditions for driving them are determined. It is found that growth rate is mainly stipulated by Rossby waves but the generation of the intense mean magnetic field is caused by Alfvén waves.

PACS numbers: 52.35.Mu, 92.10.hf, 94.20.wc

Keywords: Zonal flow, Ionospheric E-layer, Rossby – Alfvén – Khantadze waves, Nonlinear instability.

1. Introduction

Large-scale wave motions have the significant influence on energy balance in the Earth's atmospheric circulation [1, 2]. However, the presence of charged particles in the electrically conductive weakly ionized ionosphere substantially enriches the conditions for propagation of different nature low-frequency wave modes. Numerous ground-based and satellite observations [3 – 20] show that planetary-scale (with wavelengths $\lambda \geq 10^3 km$ and several days period) wave perturbations of electromagnetic (EM) origin regularly exist in different ionospheric layers. Increasing interest to the planetary-scale ultra-low-frequency (ULF) wave perturbations is caused by the fact that many ionospheric phenomena from the same frequency range can play the role of ionospheric precursors of some extraordinary phenomena

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(earthquakes, volcano eruptions, etc.) [21 – 23] and also appear as the ionospheric response to the anthropogenic activity [24 – 27]. Forced oscillations of that kind under the impulsive impacts on the ionosphere and during magnetospheric storms were also observed [21].

In recent years increasing number of theoretical and experimental investigations was devoted to the investigation of dynamics of Rossby type waves (induced by the spatial inhomogeneity of the Coriolis parameter) in the Earth's ionosphere. Dokuchaev [28] first indicated the necessity of accounting for interaction of induced electric current with the Earth's magnetic field on the winds dynamics. The next step was done by Tolstoy [29] pointed out the importance of other global factor, acting permanently in the ionosphere - space inhomogeneity of the geomagnetic field on the dynamics of Rossby type waves in the Earth's ionospheric E-layer. The waves were entitled hydromagnetic gradient (HMG) waves. It was also shown that HMG waves can couple with the Rossby waves in the E-layer heights. He suggested that HMG waves may appear as traveling perturbations of the S_q current system producing from a few to several tenths of *nT* strong variations of the geomagnetic field.

Recently, in [30 – 34] was established new type of waves propagating in the ionospheric E-layer. They can be considered as the generalization of tropospheric Rossby waves by the spatially inhomogeneous geomagnetic field $\mathbf{B}_0$. As distinct from HMG waves, these waves do not cause the Earth's magnetic field significant perturbation and are produced by the dynamo electric field $\mathbf{E}_d = \mathbf{v} \times \mathbf{B}_0$. Note that in addition these waves are caused by the Hall conductivity in the E-layer. The waves of such different from HMG waves nature were termed "magnetized Rossby (MR) waves" [32].

Both HMG and MR waves compose so called slow long-period group of planetary waves having quite low phase velocities of the order of the local ionospheric winds (1 – 100 m/s). At middle – latitudes, their wavelengths ~ 10^3 *km* and longer, but the wave period alter from 2 h to 14 days. Correspondingly, the frequency falls in the range of $10^{-4} – 10^{-6}$ s^{-1}. In the experiments [3 – 5, 9, 10, 14, 20] some characteristics of these waves are observed.

Under the space (latitudinal) inhomogeneity of the geomagnetic field and Hall effect new type of waves, so called fast large-scale EM perturbations in the middle-latitude ionosphere also can propagate. In contrast to the slow waves, the fast modes are associated with oscillations of the ionospheric electrons frozen in the geomagnetic field and are connected with the large-scale internal vortical electric field generation in the ionosphere, i.e. $\mathbf{E}_v = \mathbf{V}_D \times \mathbf{B}_0$, where $\mathbf{V}_D = \mathbf{E} \times \mathbf{B}_0 / B_0^2$ is an electron drift velocity. The fast EM waves propagate along the parallels against the mean-zonal flow to the east as well as to the west. In E-region the phase velocity of fast waves is sufficiently high $|c_B| \approx 2 – 20$ kms^{-1}. Due to the dependence of c_B on the density of the charged particles the appropriate frequency of fast waves ($\omega \approx k_x c_B$) also changes almost by one order of magnitude during daytime and nighttime. As compared to the slow waves fast modes have relatively high frequency in the range 10^{-1}- 10^{-4} s^{-1} with the corresponding periods from 4 *min* to 6 *h* and the wavelength $\geq 10^3$ *km*. In contrast to the slow modes, fast EM planetary waves give rise to strong pulsations of the geomagnetic field 20 – 80 *nT*. Such new type of large – scale ULF wave EM perturbations in the ionospheric E - and F - regions first was theoretically revealed in [35 – 37], where the first classification of the EM planetary waves into fast and slow waves also is given. Such fast EM planetary waves are called Khantadze waves and were recorded in the middle and moderate latitudes during the launching of spacecrafts [18] and fixed by the ionospheric and magnetic world network observations [9, 19, 38].

Extensive analysis of the planetary EM waves in the ionospheric E - and F - layers is given in [39 – 41]. It was shown that large-scale waves are weakly damped. New type of coupled Rossby waves with Alfvén waves first was revealed in [42], where the possibility of existence of the new spatially isolated joint Alfvén – Rossby nonlinear vortical structures in the Earth's ionosphere is also shown. We believe that the further investigation of the nonlinear dynamics of ULF planetary EM waves is so necessary.

In the given paper, we show that the action of the latitudinal inhomogeneity of both the Coriolis parameter and the geomagnetic field through the vertically propagating geomagnetic field perturbations lead to the coupled propagation of EM Rossby – Alfvén – Khantadze modes. By this fact the initial equations describing the appropriate nonlinear dynamics becomes 3D. The aim of the present paper is to investigate the possibility of mean zonal – flow and magnetic field generation by the EM coupled Rossby – Alfvén – Khantadze (CRAK) planetary waves in the ionospheric E - layer.

Ground - based and satellite observations [1, 2] confirm the permanent existence of large-scale azimuthally symmetric band – like sheared zonal flows surrounding the globe at different layers of the Earth's ionosphere and propagating along the parallels with inhomogeneous velocities along the meridians (see, e.g. [43]). Thus, the Earth's ionosphere represents the dynamical system of different nature waves and zonal flows. Under such favorable conditions for nonlinear interactions different EM nonlinear stationary solitary structures can appear [42, 44].

According to the one existing idea spatially inhomogeneous zonal winds (shear flows) can be produced by nonuniform heating of the atmospheric layers by solar radiation. First in [45] was suggested the generation mechanism of zonal flows by tropospheric Rossby waves in neutral atmosphere invoking parametric instability in terms of the kinetic equation for wave packets. The investigation of zonal – flow generation problem by Rossby waves was further developed in [46, 47] using the parametric instabilities mechanism on the basis of a monochromatic four – wave resonant nonlinear interaction. In these papers it was shown that zonal flows in a non – uniform rotating neutral atmosphere can be excited by finite – amplitude Rossby waves. Accordingly, these papers study the interaction of pump waves (Rossby waves), a sheared flow and two satellites of the pump wave (side – band waves). This approach is an alternative to the standard weak turbulence approach used by [45]. The driving mechanism of this instability is due to the Reynolds stresses, which are inevitably inherent for finite – amplitude small – scale Rossby waves. Owing to this essential nonlinear mechanism, spectral energy transfers from small – scale Rossby waves to large – scale enhanced zonal flows (inverse cascade) in the Earth's neutral atmosphere. In addition, the zonal - flow generation was considered within a simple model of Rossby wave turbulence, using the classical nonlinear two – dimensional Charney equation. It was found that the necessary condition for zonal flow generation is similar to the Lighthill criterion for modulation instability in nonlinear optics [48]. By the numerical simulation of sheared zonal flow interaction with Rossby waves in the Earth's neutral atmosphere [49] is shown that new solitary structures arise to produce the structural turbulence.

Further [50] revealed the new mechanism for the problem of zonal flow generation by the drift waves in magnetized plasmas adding a scalar nonlinearity of Korteweg – de Vries type to the generalized Hasegawa – Mima equation containing the vector nonlinearity also. It was shown that in this case zonal – flow generation always exists and needs no criterion fulfillment.

Investigation of the mean zonal flow generation problem in the Earth's electrically conducting ionosphere was firstly undertaken in [51 – 54], where the excitation of zonal flow by MR waves in the ionospheric E - layer was considered.

However, the investigation of another very important nonlinear process, viz., the generation of mean zonal flows and magnetic field by EM planetary waves in the ionospheric layers was started recently. Nonlinear dynamics of coupled Rossby – Khantadze and coupled internal – gravity and Alfvén EM planetary waves in the weakly ionized ionospheric E – layer was investigated by [55, 56]. It was shown that such EM planetary waves along with mean zonal flows can generate intense mean magnetic fields also. In the present paper, we will focus our attention on the Earth's weakly ionized, conductive ionospheric gas of the E - layer (≈ 90 – 150 km from the Earth's surface) and will consider the generation of mean zonal flow and magnetic field by coupled Rossby – Alfvén – Khantadze (CRAK) EM planetary waves. Developed in [57, 58] techniques for the case of EM waves will be used. The paper is organized as follows: In Sec. 2, basic equations modeling the nonlinear propagation of EM CRAK planetary waves in the ionospheric E - layer are obtained. Linear propagation properties of the EM coupled Rossby – Alfvén – Khantadze waves are given in detail in Sec. 3. Using the modified parametric approach, a set of coupled equations describing the nonlinear interaction of pumping EM CRAK planetary waves with an arbitrary spectrum and zonal flows is derived in Sec. 4. In the same section zonal flow dispersion relation is also obtained. In Secs. 5 and 6 it is shown that the system of equations obtained in Sec. 4 is unstable to a three wave parametric instability, whereby a coherent, monochromatic pumping Rossby – Alfvén – Khantadze waves can drive a band of modes and associated zonal flow and magnetic field generation. Namely, in Sec. 5 zonal flow growth rate is analyzed in detail. In Sec. 6, magnetic field generation dynamics is investigated in detail. Our discussion and conclusions are presented in Sec. 7.

2. Physical modeling for ionospheric E – layer

We consider the weakly ionized ionospheric E – layer plasma comprising of electrons, ions, and neutral (molecules) particles. Due to the condition $n / N << 1$, where n and N are the equilibrium number densities for the charged particles and neutrals, respectively and strong collisional coupling between the ions and neutrals the dynamics of such ionospheric E – layer gas is largely determined by its massive neutral component. Attributable by the existence of charged particles Ampere force plays the significant role in the problem set along with the effects of the latitudinal inhomogeneity of the vertical component of the Earth's angular rotation $\mathbf{\Omega}$ and of the geomagnetic field $\mathbf{B}_0(\mathbf{x})$ [55]. We also introduce the local Cartesian coordinates (x, y, z) system with the x - axis directed from the west to the east, y - axis directed from the south to the north and the z - axis along with the local vertical direction. The following relations for latitude λ and longitude ϕ are valid: $y=(\lambda-\lambda_0)R$ and $x=\phi R\cos\lambda_0$, where R is the distance from the Earth's center. In the defined local coordinate system, the components of the geomagnetic field vector are $\mathbf{B_0}=(0,B_{0y},B_{0z})=(0,B_{eq}\cos\lambda,-2B_{eq}\sin\lambda)$, where B_{eq} is the equatorial value of the geomagnetic field at a distance R from the Earth's center. As to the Earth's angular velocity $\mathbf{\Omega}$, we have $\mathbf{\Omega}=(0,\Omega_{0y},\Omega_{0z})=(0,\Omega_0\cos\lambda,\Omega_0\sin\lambda)$ [55].

According to [33], we can construct the following single-fluid momentum equation which describes the dynamics of the electrically conducting weakly ionized ionospheric E – layer plasma

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}+\frac{\nabla p}{\rho}-\frac{1}{\rho}\mathbf{j}\times\mathbf{B}+2\mathbf{\Omega}\times\mathbf{v}-\mathbf{g}=0, \qquad (1)$$

where $\mathbf{v}$ is the incompressible $(\nabla\cdot\mathbf{v}=0)$ neutral gas velocity, $\rho=Nm_N$ is the gas mass density, p is the gas pressure of the neutral gas and g is the gravitational acceleration. In Eq. (1) along with the Coriolis force the following Ampere force

$$\mathbf{F}_A=\frac{1}{\rho}\mathbf{j}\times\mathbf{B}=\frac{1}{\rho\mu_0}\nabla\times\mathbf{B}\times\mathbf{B}, \qquad (2)$$

is taken into account, where μ_0 is the permeability of free space, and $\mathbf{B}=\mathbf{B_0}+\mathbf{b}$ is the total magnetic induction.

From Eq. (1) follows the following equation for vorticity $\boldsymbol{\zeta}=\nabla\times\mathbf{v}$:

$$\frac{\partial\boldsymbol{\zeta}}{\partial t}-(\boldsymbol{\zeta}\cdot\nabla)\mathbf{v}+(\mathbf{v}\cdot\nabla)\boldsymbol{\zeta}-\frac{1}{\rho\mu_0}\left[(\mathbf{B}\cdot\nabla)\nabla\times\mathbf{B}-(\nabla\times\mathbf{B}\cdot\nabla)\mathbf{B}\right]+2\left[(\mathbf{v}\cdot\nabla)\mathbf{\Omega}-(\mathbf{\Omega}\cdot\nabla)\mathbf{v}\right]=0. \qquad (3)$$

By using the plasma conditions in the ionospheric E – layer we may simplify the generalized Ohm's law expression. First, the condition $\omega_{ci}/\nu_i \ll 1$($\omega_{ci}=eB/m_i$ is the ion cyclotron frequency, and ν_i is the ion – neutral collision frequency) allows to consider unmagnetized ions. Due to the high values of ν_i we can suppose $\mathbf{v}_i=\mathbf{v}$, which means that the ions are completely dragged by the ionospheric winds. As to electrons they are magnetized, $\omega_{ce}/\nu_e \gg 1$ (ω_{ce} is the electron cyclotron frequency and ν_e is the electron – neutral collision frequency). It means that electrons are frozen in the external magnetic field and they only experience drift perpendicular to the magnetic field, i.e $\mathbf{v}_e=\mathbf{v}_E=\mathbf{E}\times\mathbf{B}/B^2$. Under such conditions generalized Ohm's law for the ionospheric E- layer is [55]

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\frac{1}{en}\mathbf{j}\times\mathbf{B}=\frac{\rho}{en}\mathbf{F}_A, \qquad (4)$$

where the right-hand side reflects Hall effect [39]. Then from the Faraday's law $\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$ we can find the following equation for the magnetic induction $\mathbf{B}$ [55] :

$$\frac{\partial\mathbf{B}}{\partial t}+\frac{1}{en\mu_0}\left[(\mathbf{B}\cdot\nabla)\nabla\times\mathbf{B}-(\nabla\times\mathbf{B}\cdot\nabla)\mathbf{B}\right]-(\mathbf{B}\cdot\nabla)\mathbf{v}+(\mathbf{v}\cdot\nabla)\mathbf{B}=0. \qquad (5)$$

In contrast to the ordinary frozen in condition for a conducting fluid this equation contains the second term which is caused by the action of the Ampere force on the ionized plasma component (the Hall effect).

Eqs. (3) and (5) constitute our initial general equations. In the ionospheric E – layer, the large-scale wave motions are basically two – dimensional, i.e. $\mathbf{v} = (\mathrm{v}_x, \mathrm{v}_y, 0)$ and by using the incompressibility condition $\nabla \cdot \mathbf{v} = 0$, we can introduce the stream function $\psi(x, y, z)$, so that $\mathrm{v}_x = -\partial\psi/\partial y$, and $\mathrm{v}_y = \partial\psi/\partial x$. Further we will consider sufficiently high latitudes in the northern hemisphere, assuming that the geomagnetic field $\mathbf{B}_0 = B_{0z}(y)\mathbf{e}_z$ and the Earth's angular velocity $\mathbf{\Omega} = \Omega_{0z}(y)\mathbf{e}_z$. Let us suppose that the magnetic induction perturbation is also two – dimensional, i.e. $\mathbf{b} = (b_x, b_y, 0)$ and according to the condition $\nabla \cdot \mathbf{B} = 0$, we can introduce the magnetic function $A(x, y, z)$, so that $b_x = \partial A/\partial y$, and $b_y = -\partial A/\partial x$. Then from Eq. (3) we get

$$\frac{\partial \Delta_\perp \psi}{\partial t} + \beta \frac{\partial \psi}{\partial x} + J(\psi, \Delta_\perp \psi) = -\frac{B_{0z}}{\rho\mu_0} \frac{\partial \Delta_\perp A}{\partial z} + \frac{1}{\rho\mu_0} J(A, \Delta_\perp A). \tag{6}$$

Here, $\beta = \partial f/\partial y = 2\partial\Omega_{0z}/\partial y$, $\Delta_\perp = \partial_x^2 + \partial_y^2$ is the two – dimensional (2D) Laplacian and $J(a,b) = \partial_x a\, \partial_y b - \partial_y a\, \partial_x b$ is the the vector nonlinearity called Jacobian (Poisson bracket). Note that in Eq. (6) we neglected the term containing $\partial B_{0z}/\partial y$ compared with the first term on the right – hand side.

To transform magnetic induction Eq. (5) we consider its x – and y – components in terms of magnetic function A:

$$\frac{\partial^2 A}{\partial t \partial y} + \frac{B_{0z}}{en\mu_0} \frac{\partial^3 A}{\partial x \partial z^2} - c_B \frac{\partial^2 A}{\partial x \partial y} + B_{0z} \frac{\partial^2 \psi}{\partial y \partial z} + J(\psi, \frac{\partial A}{\partial y}) - J(A, \frac{\partial \psi}{\partial y}) = 0, \tag{7}$$

$$\frac{\partial^2 A}{\partial t \partial x} - \frac{B_{0z}}{en\mu_0} \frac{\partial^3 A}{\partial y \partial z^2} - c_B \frac{\partial^2 A}{\partial x^2} + B_{0z} \frac{\partial^2 \psi}{\partial x \partial z} + J(\psi, \frac{\partial A}{\partial x}) - J(A, \frac{\partial \psi}{\partial x}) = 0, \tag{8}$$

where $c_B = \beta_B / en\mu_0$, $\beta_B = \partial B_{0z}/\partial y$. In Eqs. (7), and (8) we neglected the terms ~ A^2/L^4 in comparison with $A\psi/L^3$, where L is the scale-length for planetary waves.

Let's integrate Eqs. (7), and (8) by y, and x, respectively. We get

$$\frac{\partial A}{\partial t} + \frac{1}{en\mu_0} \int dy B_{0z}(y) \frac{\partial^3 A}{\partial x \partial z^2} - c_B \frac{\partial A}{\partial x} + \int dy B_{0z}(y) \frac{\partial^2 \psi}{\partial y \partial z} + J(\psi, A) = F_1(x, z), \tag{9}$$

$$\frac{\partial A}{\partial t} - \frac{B_{0z}}{en\mu_0} \int dx \frac{\partial^3 A}{\partial z^2 \partial y} - c_B \frac{\partial A}{\partial x} + B_{0z} \frac{\partial \psi}{\partial z} + J(\psi, A) = F_2(y, z). \tag{10}$$

Here, F_1 and F_2 are arbitrary functions of integration. Let us represent in Eqs. (9) and (10) $B_{0z}(y) \approx B_{0z}(y_0) + y\,\partial B_{0z}/\partial y$, then we get

$$\begin{aligned} &\frac{\partial A}{\partial t} + \frac{B_{0z}(y_0)}{en\mu_0} \int dy \frac{\partial^3 A}{\partial x \partial z^2} + c_B \int dy\, y \frac{\partial^3 A}{\partial x \partial z^2} - c_B \frac{\partial A}{\partial x} + B_{0z}(y_0) \frac{\partial \psi}{\partial z} \\ &+ \frac{\partial B_{0z}}{\partial y} \int dy\, y \frac{\partial^2 \psi}{\partial y \partial z} + J(\psi, A) = F_1(x, z), \end{aligned} \tag{11}$$

$$\frac{\partial A}{\partial t} - \frac{B_{0z}}{en\mu_0} \int dx \frac{\partial^3 A}{\partial z^2 \partial y} - c_B \frac{\partial A}{\partial x} + B_{0z}(y_0) \frac{\partial \psi}{\partial z} + J(\psi, A) = F_2(y, z). \tag{12}$$

For the consistency of Eqs. (11), and (12), we choose:

$$F_1(x, z) = \frac{B_{0z}(y_0)}{en\mu_0} \int dy \frac{\partial^3 A}{\partial x \partial z^2} + c_B \int dy\, y \frac{\partial^3 A}{\partial x \partial z^2} + \frac{\partial B_{0z}}{\partial y} \int dy\, y \frac{\partial^2 \psi}{\partial y \partial z}, \tag{13}$$

$$F_2(y,z) = -\frac{B_{0z}(y_0)}{en\mu_0}\int dx \frac{\partial^3 A}{\partial z^2 \partial y}. \tag{14}$$

Then we get the following common equation

$$\frac{\partial A}{\partial t} - c_B \frac{\partial A}{\partial x} + B_{0z}(y_0)\frac{\partial \psi}{\partial z} + J(\psi, A) = 0. \tag{15}$$

Equations (6) and (15) compose the initial system of equations for our problem and describe the nonlinear dynamics of the EM planetary low – frequency wave perturbations in the ionospheric E – layer. From Eqs. (6) and (15), we can obtain the following temporal conservation law of energy $\mathcal{E}$

$$\frac{\partial \mathcal{E}}{\partial t} = \frac{\partial}{\partial t}\left\{\frac{1}{2}\int [\rho(\nabla_\perp \psi)^2 + \frac{1}{\mu_0}(\nabla_\perp A)^2] dxdy\right\} = 0. \tag{16}$$

3. Linear EM planetary waves

Linear dispersion relation for EM CRAK waves can be readily obtained from Eqs. (6) and (15)

$$(\omega + \frac{k_x}{k_\perp^2}\beta)(\omega + k_x c_B) = k_z^2 \mathrm{v}_A^2, \tag{17}$$

where ω is the wave frequency, $\mathrm{v}_A^2 = B_{0z}^2 / \mu_0 \rho$ is the squared Alfvén velocity, and $k_\perp^2 = k_x^2 + k_y^2$, k_x, k_y, and k_z are the components of the wave vector $\mathbf{k}$ along the x-, y-, and z- axes. When $k_x = 0$ we get the Alfvén branch of oscillations with the dispersion relation $\omega = \pm k_z \mathrm{v}_A$; when $k_z = 0$ we get the additional two branches of oscillations: 1) $\omega = -k_x \beta / k_\perp^2$, which describes the Rossby waves (slow waves), and 2) $\omega = -k_x c_B$, which describes the Khantadze waves (fast waves). Thus the dispersion relation (17) describes the propagation of EM CRAK waves in the ionospheric E – layer.

The solution of the dispersion equation (17) by taking into account the velocity $c_B < 0$

$$\omega_{1,2} = \frac{k_x}{2}\left[|c_B| - \frac{\beta}{k_\perp^2} \pm \sqrt{\left(|c_B| + \frac{\beta}{k_\perp^2}\right)^2 + 4\frac{k_z^2}{k_x^2}\mathrm{v}_A^2}\right]. \tag{18}$$

Eq. (18) represents that EM coupled Rossby – Alfvén – Khantadze waves have two branches of oscillations, one branch of oscillation ω_1 (with “+” sign before the radical) and other one ω_2 (with “–” sign before the radical).

Eq. (18) for the case of small $k_\perp^2 << 1$ reads as follows

$$\omega_1 = k_x(|c_B| + k_\perp^2 \frac{k_z^2 \mathrm{v}_A^2}{\beta k_x^2}), \tag{19}$$

and

$$\omega_2 = k_x(-\frac{\beta}{k_\perp^2} - k_\perp^2 \frac{k_z^2 \mathrm{v}_A^2}{\beta k_x^2}). \tag{20}$$

As to the case of large $k_\perp^2 >> 1$, we get from Eq. (18)

$$\omega_{1,2} = \frac{k_x}{2}\left\{|c_B| - \frac{\beta}{k_\perp^2} \pm \sqrt{c_B^2 + 4\frac{k_z^2}{k_x^2}\mathrm{v}_A^2}\left[1 + \frac{|c_B|\beta}{k_\perp^2\left(c_B^2 + 4\frac{k_z^2}{k_x^2}\mathrm{v}_A^2\right)}\right]\right\}. \tag{21}$$

Here we consider $k_z^2 \mathrm{v}_A^2 / k_x^2 c_B^2 \sim 1$ to obtain Eqs. (19) – (21).

Thus the branch ω_1 reprsents the Khantadze waves imposed by the action of both the latitudinal inhomogeneity of the Coriolis force and magnetic field perturbations, while the branch ω_2 represents the Rossby waves imposed by the same factors. Under the action of these factors Khantadze waves are propagating eastward with the increased phase velocity ω_1 / k_x, while the phase velocity of westward propagating Rossby waves is also increasing.

The case of small $k_x^2 << 1$, also can be described from Eq. (18)

$$\omega_{1,2} = \pm k_z \mathrm{v}_A \left[1 + k_x^2 \frac{\left(|c_B| + \frac{\beta}{k_\perp^2} \right)^2}{8 k_z^2 \mathrm{v}_A^2} \right] + \frac{k_x}{2} \left(|c_B| - \frac{\beta}{k_\perp^2} \right). \quad (22)$$

These are Alfvén waves branch imposed by the action of latitudinal inhomogeneity of Coriolis force and latitudinal inhomogeneity of the geomagnetic field.

We can represent in the β - plane approximation [33] the Coriolis parameter as

$$f = 2\Omega_{0z} = 2\Omega_0 \sin\lambda = f_0 + \beta y, \quad (23)$$

with

$$\beta = \frac{\partial f}{\partial y} = \frac{2\Omega_0 \cos\lambda_0}{R} > 0, \quad (24)$$

and the geomagnetic field as

$$B_{0z} = -2B_{eq} \sin\lambda = \gamma_0 + \beta_B y, \quad (25)$$

with

$$\beta_B = \frac{\partial B_{0z}}{\partial y} = -\frac{2B_{eq} \cos\lambda_0}{R} < 0. \quad (26)$$

By introducing the dimensionless variables $k^* = k|c_B|^{1/2} / \beta^{1/2}$, and $\omega^* = \omega / \beta^{1/2} |c_B|^{1/2}$, we can rewrite the dispersion relation (18) as

$$y_{1,2} = \frac{1}{2x^2} \left(x^2 - 1 \pm \sqrt{(x^2+1)^2 + 4x^4 \alpha} \right), \quad (27)$$

where $y = \omega^* / k_x^*$, $x^2 = k_\perp^{*2}$, and $\alpha = k_z^2 \mathrm{v}_A^2 / k_x^2 c_B^2$. For the ionospheric E – layer parameters $B_{eq} \sim 0.5\times 10^{-4} T$, $2\Omega_0 \sim 10^{-4} rad/s$, $n/N \sim 10^{-8} - 10^{-6}$, $\rho = (10^{-7} - 10^{-8}) kgm^{-3}$, we can find that $|c_B| \sim (1-10) km/s$, $\mathrm{v}_A \sim (0.1-1) km/s$. In Fig. 1, the dependence of dimensionless phase velocity y of coupled Rossby – Alfvén – Khantadze branches of oscillations on wave number x for the different values of $\alpha = 0$; 1; 5 is shown. A and B curves correspond to "+" and "–" signs before the radical in Eq. (27), respectively. Thus A and B curves correspond to ω_1 and ω_2 branches of oscillations in Eqs. (19) – (21), respectively.

We can find the following behavior of $y_{1,2}$:

a) when $x \to 0$,

$$y_1 = 1 + x^2 \alpha, \quad \text{and} \quad y_2 = -\frac{1}{x^2} - x^2 \alpha. \quad (28)$$

b) when $x \to \infty$

$$y_{1,2}=\frac{1}{2}\left[1-\frac{1}{x^2}\pm\left(\sqrt{1+4\alpha}+\frac{1}{x^2\sqrt{1+4\alpha}}\right)\right]. \qquad (29)$$

In Eqs. (28) and (29) y_1 and y_2 correspond to Khantadze and Alfvén waves, respectively.

4. Nonlinear interaction of coupled Rossby – Alfvén – Khantadze EM planetary waves and zonal flow dispersion relation

To find the possibility for the zonal flow generation by the EM CRAK planetary waves in the ionospheric E – layer we will consider the initial nonlinear Eqs. (6) and (15). Existing in this equations the nonlinear Jacobian terms allows to consider a standard four - wave nonlinear interaction, in which the coupling between the pump $\tilde{X}=(\tilde{\psi},\tilde{h})$ EM planetary waves and two side - band $\hat{X}=(\hat{\psi},\hat{h})$ modes drives low - frequency large - scale $\overline{X}=(\overline{\psi},\overline{h})$ zonal flows with variation only along the y – axis. Accordingly, the total perturbed quantities $X=(\psi,h)$ are decomposed in three components,

$$X=\tilde{X}+\hat{X}+\overline{X}, \qquad (30)$$

where

$$\tilde{X}=\sum_{\mathbf{k}}\left[\tilde{X}_{+}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)+\tilde{X}_{-}(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{r}+i\omega_{\mathbf{k}}t)\right], \qquad (31)$$

describes pump EM planetary modes spectrum ($\tilde{X}_{-}(\mathbf{k})=\tilde{X}^{*}_{+}(\mathbf{k})$, where $*$ means the complex conjugate),

$$\hat{X}=\sum_{\mathbf{k}}\left[\hat{X}_{+}(\mathbf{k})\exp(i\mathbf{k}_{+}\cdot\mathbf{r}-i\omega_{\mathbf{k}_{+}}t)+\hat{X}_{-}(\mathbf{k})\exp(i\mathbf{k}_{-}\cdot\mathbf{r}-i\omega_{\mathbf{k}_{-}}t)+c.c.\right] \qquad (32)$$

describes sideband modes spectrum, and

$$\overline{X}=\overline{X}_0\exp(-i\Omega t+iq_y y)+c.c. \qquad (33)$$

describes the zonal - flow modes varying only along meridians. Within the local approximation the amplitude of the zonal flow mode $\overline{X}_0=(\overline{\psi}_0,\overline{h}_0)$ is assumed constant. The energy and momentum conservations $\omega_{\pm}=\Omega\pm\omega_{\mathbf{k}}$ and $\mathbf{k}_{\pm}=q_y\mathbf{e}_{\mathbf{y}}\pm\mathbf{k}$ are fulfilled, and the pairs $(\omega_{\mathbf{k}},\mathbf{k})$ and $(\mathbf{\Omega},q_y\mathbf{e}_y)$ represent the frequency and wave vector of the EM planetary pump and zonal - flow modes, respectively. In the sequel we will omit the index $\mathbf{k}$ at ω for simplicity.

Substituting Eqs. (30) - (33) into (6) and (15), and according to the standard quasilinear procedure ignoring the small nonlinear term in the relations for the high frequency but not for the low frequency zonal flow modes we get for the EM planetary modes

$$\begin{cases}(\omega k_{\perp}^2+\beta k_x)\tilde{\psi}_{\pm}=\dfrac{B_{0z}}{\rho\mu_0}k_z k_{\perp}^2\tilde{A}_{\pm},\\(\omega+c_B k_x)\tilde{A}_{\pm}=B_{0z}k_z\tilde{\psi}_{\pm}.\end{cases} \qquad (34)$$

From this homogeneous system the dispersion relation (17) for EM planetary modes follows.

Substituting Eqs. (30) – (33) into (6) and (15) to obtain the relations for the amplitude of the zonal flow modes and averaging out over the fast small – scale fluctuations, we get [57, 58]

$$-i\Omega\overline{\psi}_0=R_{\perp}, \qquad (35)$$

and

$$-i\Omega\overline{A}_0=R_{\parallel}, \qquad (36)$$

where $R_{\perp}$ and $R_{\parallel}$ are the mixture of Reynolds and electromotive forces, defined by

$$R_{\perp} = -\left\langle \frac{\partial\tilde{\psi}}{\partial x}\frac{\partial\hat{\psi}}{\partial y} + \frac{\partial\hat{\psi}}{\partial x}\frac{\partial\tilde{\psi}}{\partial y} \right\rangle + \frac{1}{\rho\mu_0}\left\langle \frac{\partial\tilde{A}}{\partial x}\frac{\partial\hat{A}}{\partial y} + \frac{\partial\hat{A}}{\partial x}\frac{\partial\tilde{A}}{\partial y} \right\rangle, \tag{37}$$

and

$$R_{\parallel} = iq_y \left\langle \tilde{\psi}\frac{\partial\hat{A}}{\partial x} + \hat{\psi}\frac{\partial\tilde{A}}{\partial y} \right\rangle, \tag{38}$$

where $\langle\cdots\rangle$ represents the average over fast oscillations. Using the Fourier series (31) and (32), we can write these quantities as

$$R_{\perp} = -\sum_{\mathbf{k}} k_x r_{\perp}(\mathbf{k}), \tag{39}$$

and

$$R_{\parallel} = q_y \sum_{\mathbf{k}} k_x r_{\parallel}(\mathbf{k}), \tag{40}$$

where

$$\begin{aligned} r_{\perp}(\mathbf{k}) &= q_y(\tilde{\psi}_-\hat{\psi}_+ - \tilde{\psi}_+\hat{\psi}_-) + 2k_y(\tilde{\psi}_-\hat{\psi}_+ + \tilde{\psi}_+\hat{\psi}_-) \\ &\quad - \frac{1}{\rho\mu_0}[q_y(\tilde{A}_-\hat{A}_+ - \tilde{A}_+\hat{A}_-) + 2k_y(\tilde{A}_-\hat{A}_+ + \tilde{A}_+\hat{A}_-)], \end{aligned} \tag{41}$$

and

$$r_{\parallel}(\mathbf{k}) = \tilde{\psi}_+\hat{A}_- - \tilde{\psi}_-\hat{A}_+ + \hat{\psi}_+\tilde{A}_- - \hat{\psi}_-\tilde{A}_+ = \tilde{\psi}_-\hat{\lambda}_+ - \tilde{\psi}_+\hat{\lambda}_-. \tag{42}$$

Here we used Eq. (34) for $\tilde{A}_{\pm}$ to construct the following auxiliary side – band amplitudes

$$\hat{\lambda}_{\pm} = \frac{k_z B_{0z}}{\omega + k_x c_B}\hat{\psi}_{\pm} - \hat{A}_{\pm}. \tag{43}$$

To calculate the functions $r_{\perp}$ and $r_{\parallel}$, we need to define the side – band amplitudes $\hat{\psi}_{\pm}$ and $\hat{A}_{\pm}$. According to Eqs. (6) and (15), these amplitudes satisfy the following system [57, 58]

$$\begin{cases} (\omega_{\pm}k_{\perp\pm}^2 \pm k_x\beta)\hat{\psi}_{\pm} \mp k_z k_{\perp\pm}^2 \dfrac{B_{0z}}{\rho\mu_0}\hat{A}_{\pm} = \mp ik_x q_y(k_{\perp}^2 - q_y^2)\tilde{\psi}_{\pm}\bar{\psi}_0 \\ \qquad \pm i\dfrac{q_y}{\rho\mu_0}(k_{\perp}^2 - q_y^2)\dfrac{k_x k_z B_{0z}}{\omega + k_x c_B}\tilde{\psi}_{\pm}\bar{A}_0, \\ \pm k_z B_{0z}\hat{\psi}_{\pm} - (\omega_{\pm} \pm k_x c_B)\hat{A}_{\pm} = \pm ik_x q_y\tilde{\psi}_{\pm}\bar{A}_0\left(\dfrac{k_z B_{0z}}{\omega + k_x c_B}\dfrac{\bar{\psi}_0}{\bar{A}_0} - 1\right). \end{cases} \tag{44}$$

We can find the following solutions of the system (44)

$$\begin{aligned} \hat{\psi}_{\pm} = \frac{ik_x q_y}{D_{\pm}}\tilde{\psi}_{\pm}\Bigg\{&\bar{\psi}_0\left[\mp(k_{\perp}^2 - q_y^2)(\omega_{\pm} \pm k_x c_B) - \frac{k_z^2 \mathrm{v}_A^2 k_{\perp\pm}^2}{\omega + k_x c_B}\right] \\ &+ \bar{A}_0\frac{k_z B_{0z}}{\rho\mu_0}\left[\pm\frac{k_{\perp}^2 - q_y^2}{\omega + k_x c_B}(\omega_{\pm} \pm k_x c_B) + k_{\perp\pm}^2\right] \end{aligned} \tag{45}$$

and

$$\hat{A}_{\pm} = \pm \frac{ik_x q_y}{D_{\pm}} \tilde{\psi}_{\pm} \left\{ \bar{\psi}_0 \left[-\frac{k_z B_{0z}}{\omega + k_x c_B} (\omega_{\pm} k_{\perp\pm}^2 \pm k_x \beta) \mp k_z B_{0z} (k_\perp^2 - q_y^2) \right] \right.$$
$$\left. + \bar{A}_0 \left[\omega_{\pm} k_{\perp\pm}^2 \pm k_x \beta \pm \frac{k_\perp^2 - q_y^2}{\omega + k_x c_B} k_z^2 \mathrm{v}_A^2 \right] \right\}, \tag{46}$$

where

$$D_{\pm} = (\omega_{\pm} k_{\perp\pm}^2 \pm k_x \beta)(\omega_{\pm} \pm k_x c_B) - k_z^2 \mathrm{v}_A^2 k_{\perp\pm}^2 . \tag{47}$$

Applying Eqs. (45) and (46), the expression (43) for the auxiliary side – band amplitudes takes the following form

$$\hat{\lambda}_{\pm} = \frac{ik_x q_y}{D_{\pm}} \tilde{\psi}_{\pm} \bar{A}_0 \left\{ \mp \frac{\Omega (k_\perp^2 - q_y^2)}{\omega + k_x c_B} \left[\frac{\bar{\psi}_0}{\bar{A}_0} k_z B_{0z} - \frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B} \right] \right.$$
$$\left. + \left(1 - \frac{k_z B_{0z}}{\omega + k_x c_B} \frac{\bar{\psi}_0}{\bar{A}_0} \right) \left[-2 q_y \Omega k_y + \frac{k_x}{k_\perp^2} \beta q_y^2 \pm 2 q_y k_y \beta \frac{k_x}{k_\perp^2} \mp \Omega (k_\perp^2 + q_y^2) \right] \right\}. \tag{48}$$

We assume that $q_y / k_\perp \sim \Omega / \omega << 1$, which is valid in the existing theory of zonal – flow generation [45]. Then, from Eq. (48) follows the distinguished fact that the main contributions of the "magnetic" and "stream function" side – band amplitudes to the evolution equation of the mean magnetic field mutually cancel each other [see Eqs. (36), (38), (40), and (42)]. If we use the superscripts "(1), (2), …" to show the order of magnitudes with respect to q_y and Ω, then Eq. (47) can be written as follows

$$D_{\pm} = \pm D^{(1)} + D^{(2)} \pm D^{(3)} + D^{(4)}, \tag{49}$$

where

$$D^{(1)} = 2 q_y k_y \omega (\omega + k_x c_B) + (\omega + k_x c_B) k_\perp^2 \Omega + \Omega (\omega k_\perp^2 + k_x \beta) - 2 q_y k_y k_z^2 \mathrm{v}_A^2,$$
$$D^{(2)} = -q_y^2 k_z^2 \mathrm{v}_A^2 + \Omega^2 k_\perp^2 + 2 q_y k_y \Omega (\omega + k_x c_B) + \omega q_y^2 (\omega + k_x c_B) + 2 q_y k_y \omega \Omega,$$
$$D^{(3)} = 2 q_y k_y \Omega^2 + \Omega q_y^2 (\omega + k_x c_B) + \omega \Omega q_y^2,$$
$$D^{(4)} = \Omega^2 q_y^2. \tag{50}$$

If we keep in Eq. (48) only first two terms over the named above small parameters q_y and Ω we get

$$\hat{\lambda}_{\pm} = \hat{\lambda}_{\pm}^{(1)} + \hat{\lambda}_{\pm}^{(2)}, \tag{51}$$

where

$$\hat{\lambda}_{\pm}^{(1)} = -i \frac{k_x q_y}{D^{(1)}} \tilde{\psi}_{\pm} \bar{A}_0 \left[\frac{\Omega k_\perp^2}{\omega + k_x c_B} \left(\frac{\bar{\psi}_0}{\bar{A}_0} k_z B_{0z} - \frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B} \right) \right.$$
$$\left. + \left(1 - \frac{k_z B_{0z}}{\omega + k_x c_B} \frac{\bar{\psi}_0}{\bar{A}_0} \right) \left(-2 q_y k_y \beta \frac{k_x}{k_\perp^2} + \Omega k_\perp^2 \right) \right], \tag{52}$$

$$\hat{\lambda}_{\pm}^{(2)} = \pm i \frac{k_x q_y}{D^{(1)}} \tilde{\psi}_{\pm} \bar{A}_0 \left[\frac{D^{(2)}}{D^{(1)}} \frac{\Omega k_\perp^2}{\omega + k_x c_B} \left(\frac{\bar{\psi}_0}{\bar{A}_0} k_z B_{0z} - \frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B} \right) \right.$$
$$\left. + \left(1 - \frac{k_z B_{0z}}{\omega + k_x c_B} \frac{\bar{\psi}_0}{\bar{A}_0} \right) \left(-2 q_y \Omega k_y + \frac{k_x}{k_\perp^2} \beta q_y^2 - 2 k_y q_y \frac{k_x}{k_\perp^2} \beta \frac{D^{(2)}}{D^{(1)}} + \Omega k_\perp^2 \frac{D^{(2)}}{D^{(1)}} \right) \right]. \tag{53}$$

We can prove that the contribution of $\hat{\lambda}_{\pm}^{(1)}$ in Eq. (42) is zero. Thus, we get

$$r_{\|}(\mathbf{k}) = \tilde{\psi}_{-}\hat{\lambda}_{+}^{(2)} - \tilde{\psi}_{+}\hat{\lambda}_{-}^{(2)}, \tag{54}$$

where for $\hat{\lambda}_{\pm}^{(2)}$ we can obtain

$$\hat{\lambda}_{\pm}^{(2)} = \pm i\frac{k_x q_y}{D^{(1)2}}\tilde{\psi}_{\pm}\bar{A}_0\Omega\left\{D^{(2)}\frac{k_\perp^2}{\omega + k_x c_B}\left(\frac{\bar{\psi}_0}{\bar{A}_0}k_z B_{0z} - \frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right)\right.$$
$$\left. + \left(1 - \frac{k_z B_{0z}}{\omega + k_x c_B}\frac{\bar{\psi}_0}{\bar{A}_0}\right)\left[-4k_x\beta k_y q_y\Omega + q_y^2\frac{k_x\beta}{k_\perp^2}(\omega k_\perp^2 + k_x\beta - 4k_y^2\omega) + \Omega^2 k_\perp^4\right]\right\}. \tag{55}$$

Consequently, we can transform Eq. (54) to

$$r_{\|}(\mathbf{k}) = i\frac{k_x q_y\Omega}{D^{(1)2}}I_{\mathbf{k}}(f_{\|}^{\psi}\bar{\psi}_0 + f_{\|}^{A}\bar{A}_0), \tag{56}$$

where

$$f_{\|}^{\psi} = \frac{k_z B_{0z}}{\omega + k_x c_B}\left\{-k_x\beta q_y^2\left(2\omega + k_x c_B + \frac{k_x\beta}{k_\perp^2}\right.\right.$$
$$\left.\left. - 4\frac{k_y^2}{k_\perp^2}\omega\right) + 2k_y q_y\Omega[2k_x\beta + k_\perp^2\omega + k_\perp^2(\omega + k_x c_B)]\right\}, \tag{57}$$

$$f_{\|}^{A} = \frac{1}{\omega + k_x c_B}\left\{\Omega^2 k_\perp^4\left(k_x c_B - \frac{k_x\beta}{k_\perp^2}\right)\right.$$
$$+ q_y\Omega k_y\left[-4k_x\beta(\omega + k_x c_B) - 2k_\perp^2 k_z^2 \mathrm{v}_A^2 - 2\omega k_\perp^2\left(\omega + \frac{k_x\beta}{k_\perp^2}\right)\right]$$
$$\left. + q_y^2\frac{k_x\beta}{k_\perp^2}(\omega + k_x c_B)\left[\omega k_\perp^2 + k_x\beta - 4k_y^2\omega + k_\perp^2\left(\omega + \frac{k_x\beta}{k_\perp^2}\right)\right]\right\}. \tag{58}$$

In Eq. (56) we introduced the intensity of pumping waves

$$I_{\mathbf{k}} = 2\tilde{\psi}_{+}\tilde{\psi}_{-}. \tag{59}$$

Analogously we can transform Eq. (41). To this end, we represent the solution (45) as the expansion

$$\hat{\psi}_{\pm} = \hat{\psi}_{\pm}^{(0)} + \hat{\psi}_{\pm}^{(1)}, \tag{60}$$

where

$$\hat{\psi}_{\pm}^{(0)} = \pm\frac{ik_x q_y k_\perp^2}{D^{(1)}}\tilde{\psi}_{\pm}\left\{-\bar{\psi}_0\left[(\omega + k_x c_B) + \frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right] + 2\bar{A}_0\frac{k_z B_{0z}}{\rho\mu_0}\right\}, \tag{61}$$

$$\hat{\psi}_{\pm}^{(1)} = \frac{ik_x q_y}{D^{(1)}}\tilde{\psi}_{\pm}\left\{\bar{\psi}_0\left[-k_\perp^2\Omega - 2q_y k_y\frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B} + \frac{D^{(2)}}{D^{(1)}}k_\perp^2\left(\omega + k_x c_B + \frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right)\right]\right.$$
$$\left. + \bar{A}_0\frac{k_z B_{0z}}{\rho\mu_0}\left(\frac{k_\perp^2\Omega}{\omega + k_x c_B} + 2q_y k_y - 2k_\perp^2\frac{D^{(2)}}{D^{(1)}}\right)\right\}. \tag{62}$$

Similarly for Eq. (46) we have the expansion

$$\hat{A}_{\pm} = \hat{A}_{\pm}^{(0)} + \hat{A}_{\pm}^{(1)}, \tag{63}$$

where

$$\hat{A}_{\pm}^{(0)} = \pm i\frac{k_x q_y}{D^{(1)}}\tilde{\psi}_{\pm}\left[-\overline{\psi}_0 k_z B_{0z}\left(\frac{\omega k_\perp^2 + k_x\beta}{\omega + k_x c_B} + k_\perp^2\right)\right.$$
$$\left. + \overline{A}_0\left(\omega k_\perp^2 + k_x\beta + \frac{k_\perp^2 k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right)\right], \quad (64)$$

$$\hat{A}_{\pm}^{(1)} = i\frac{k_x q_y}{D^{(1)}}\tilde{\psi}_{\pm}\left\{\overline{\psi}_0 k_z B_{0z}\left[-\frac{\Omega k_\perp^2 + 2q_y k_y\omega}{\omega + k_x c_B} + \frac{D^{(2)}}{D^{(1)}}\left(\frac{\omega k_\perp^2 + k_x\beta}{\omega + k_x c_B} + k_\perp^2\right)\right]\right.$$
$$\left. + \overline{A}_0\left[\Omega k_\perp^2 + 2q_y k_y\omega - \frac{D^{(2)}}{D^{(1)}}\left(\omega k_\perp^2 + k_x\beta + \frac{k_\perp^2 k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right)\right]\right\}. \quad (65)$$

Accordingly, for Eq. (41) we get

$$r_\perp(\mathbf{k}) = \frac{ik_x q_y}{D^{(1)^2}} I_{\mathbf{k}}\,(f_\perp^\psi \overline{\psi}_0 + f_\perp^A \overline{A}_0), \quad (66)$$

where

$$f_\perp^\psi = \frac{8k_y^3 k_z^2 \mathrm{v}_A^2 q_y^2 k_x c_B}{\omega + k_x c_B}\left(-\omega + \frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right) + q_y\Omega\left[8k_y^2\omega k_\perp^2\frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right.$$
$$+ 4k_y^2 k_\perp^2(\omega + k_x c_B)^2 - k_\perp^4(\omega + k_x c_B)^2 - k_\perp^4 k_z^2 \mathrm{v}_A^2 - 12k_y^2(\omega k_\perp^2 + k_x\beta)\frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B} \quad (67)$$
$$\left. + k_z^2 \mathrm{v}_A^2\frac{(\omega k_\perp^2 + k_x\beta)^2}{(\omega + k_x c_B)^2} + k_\perp^2 k_z^2 \mathrm{v}_A^2\frac{\omega k_\perp^2 + k_x\beta}{\omega + k_x c_B}\right],$$

$$f_\perp^A = \frac{k_z B_{0z}}{\rho\mu_0}\left\{8k_y^3 q_y^2\left[\omega k_x c_B - k_z^2 \mathrm{v}_A^2 + \frac{\omega k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B}\right]\right.$$
$$+ q_y\Omega\left[4k_y^2\omega k_\perp^2\frac{k_z^2 \mathrm{v}_A^2}{(\omega + k_x c_B)^2} - 2k_\perp^2 k_z^2 \mathrm{v}_A^2\frac{\omega k_\perp^2 + k_x\beta}{(\omega + k_x c_B)^2} + 2k_\perp^4(\omega + k_x c_B)\right.$$
$$\left. - 4k_y^2 k_\perp^2(\omega + k_x c_B) + 4k_y^2 k_\perp^2\frac{k_z^2 \mathrm{v}_A^2}{\omega + k_x c_B} - 12k_\perp^2 k_y^2\omega + 8k_y^2(\omega k_\perp^2 + k_x\beta)\right] \quad (68)$$
$$\left. + 4\Omega^2 k_\perp^2 k_y k_x\frac{\beta - k_\perp^2 c_B}{\omega + k_x c_B}\right\}.$$

Using Eqs. (39), (40), (56), and (66), we can reduce Eqs. (35) and (36) to the following form:

$$\begin{cases}\overline{\psi}_0 = I_\perp^\psi \overline{\psi}_0 + I_\perp^A \overline{A}_0, \\ \overline{A}_0 = I_\parallel^\psi \overline{\psi}_0 + I_\parallel^A \overline{A}_0,\end{cases} \quad (69)$$

where

$$I_\perp^\psi = \sum_{\mathbf{k}} \frac{k_x^2 q_y I_{\mathbf{k}}}{\Omega D^{(1)2}} \left\{ 8\frac{k_x^2 k_y^3}{k_\perp^2} k_z^2 \mathrm{v}_A^2 q_y^2 \frac{\beta c_B}{\omega + k_x c_B} \right.$$

$$+ q_y \Omega \left[(\omega + k_x c_B)^2 k_\perp^2 (4k_y^2 - k_\perp^2) + \left(1 - 12\frac{k_y^2}{k_\perp^2}\right)(\omega k_\perp^2 + k_x \beta)^2 \right. \tag{70}$$

$$\left.\left. - k_\perp^4 k_z^2 \mathrm{v}_A^2 + 8k_y^2 \omega(\omega k_\perp^2 + k_x\beta) + \frac{(\omega k_\perp^2 + k_x\beta)^3}{k_\perp^2(\omega + k_x c_B)} \right]\right\},$$

$$I_\perp^A = \sum_{\mathbf{k}} \frac{k_x^2 q_y}{\Omega D^{(1)2}} I_{\mathbf{k}} \frac{k_z B_{0z}}{\rho\mu_0} \left\{ -8k_y^3 q_y^2 \frac{k_x^2 \beta c_B}{k_\perp^2} \right.$$

$$+ q_y \Omega \left[4k_y^2 \omega \frac{\omega k_\perp^2 + k_x\beta}{\omega + k_x c_B} + 2(\omega + k_x c_B) k_\perp^2 (k_\perp^2 - 2k_y^2) + 12 k_y^2 k_x \beta \right. \tag{71}$$

$$\left.\left. -2\frac{(\omega k_\perp^2 + k_x\beta)^2}{\omega + k_x c_B} \right] + 4\Omega^2 k_\perp^2 k_y \left(-k_\perp^2 + \frac{\omega k_\perp^2 + k_x\beta}{\omega + k_x c_B} \right) \right\},$$

$$I_\parallel^\psi = -\sum_{\mathbf{k}} \frac{k_x^2 q_y^2}{D^{(1)2}} I_{\mathbf{k}} \frac{k_z B_{0z}}{\omega + k_x c_B} \left\{ -k_x \beta q_y^2 \left(2\omega + k_x c_B + \frac{k_x \beta}{k_\perp^2} - 4\frac{k_y^2}{k_\perp^2}\omega \right) \right. \tag{72}$$

$$\left. + 2k_y q_y \Omega \left[2k_x\beta + k_\perp^2(2\omega + k_x c_B) \right] \right\},$$

$$I_\parallel^A = -\sum_{\mathbf{k}} \frac{k_x^2 q_y^2}{D^{(1)2}} I_{\mathbf{k}} \frac{1}{\omega + k_x c_B} \left\{ \Omega^2 k_\perp^4 \left[\omega + k_x c_B - \frac{1}{k_\perp^2}(\omega k_\perp^2 + k_x\beta) \right] \right.$$

$$- q_y \Omega k_y \left[4k_x\beta(\omega + k_x c_B) + 2k_\perp^2 k_z^2 \mathrm{v}_A^2 + 2\omega(\omega k_\perp^2 + k_x\beta) \right] \tag{73}$$

$$\left. + 2q_y^2 \frac{k_x\beta}{k_\perp^2}(\omega + k_x c_B)(\omega k_\perp^2 + k_x\beta - 2k_y^2\omega) \right\}.$$

In Eqs. (70) – (73)

$$D^{(1)} = (\Omega - q_y V_g)\left[k_\perp^2(\omega + k_x c_B) + \omega k_\perp^2 + k_x\beta \right], \tag{74}$$

where V_g is the zonal – flow group velocity given by

$$V_g = \frac{\partial\omega}{\partial k_y} = 2\frac{k_x \beta k_y(\omega + k_x c_B)}{k_\perp^2(2\omega k_\perp^2 + k_\perp^2 k_x c_B + k_x\beta)}. \tag{75}$$

From the system of Eqs. (69), we get the following zonal – flow dispersion relation:

$$1 - (I_\perp^\psi + I_\parallel^A) + I_\perp^\psi I_\parallel^A - I_\perp^A I_\parallel^\psi = 0. \tag{76}$$

Further we will show that in the most interesting case this biquadratic with respect to $\Omega - q_y V_g$ zonal flow dispersion relation can be reduced to a quadratic one.

Let us consider the monochromatic wave packet case of the primary modes, which means a single wave vector on the right – hand sides of Eqs. (70) – (73). Because the values $I_\parallel^\psi$ and $I_\parallel^A$ are of the order of $O(q_y^2)$, while $I_\perp^\psi$ and $I_\perp^A$ are of $O(1)$, we conclude that the right – hand sides of these equations will match only if the value $\Omega - q_y V_g$ is also a small parameter. Therefore the zonal – flow dispersion relation (76) reduces to

$$1 = I_\perp^\psi, \tag{77}$$

or,

$$(\Omega - q_y V_g)^2 = -\Gamma^2, \tag{78}$$

where Γ^2 means the squared zonal – flow growth rate defined by

$$\Gamma^2 = -\frac{k_x^2 q_y^2 I_{\mathbf{k}}}{(\omega + k_x c_B)\left[(2\omega + k_x c_B)k_\perp^2 + k_x\beta\right]}\Bigg\{4k_x k_y^2 k_z^2 \mathrm{v}_A^2 \frac{c_B}{\omega + k_x c_B}$$
$$+\frac{(\omega + k_x c_B)}{\left[(2\omega + k_x c_B)k_\perp^2 + k_x\beta\right]}\Bigg[k_\perp^2(\omega + k_x c_B)^2(4k_y^2 - k_\perp^2) - 12\frac{k_y^2}{k_\perp^2}(\omega k_\perp^2 + k_x\beta)^2 \tag{79}$$
$$-k_\perp^4 k_z^2 \mathrm{v}_A^2 + 8k_y^2\omega(\omega k_\perp^2 + k_x\beta) + (\omega k_\perp^2 + k_x\beta)^2 + \frac{(\omega k_\perp^2 + k_x\beta)^3}{k_\perp^2(\omega + k_x c_B)}\Bigg]\Bigg\}.$$

5. Generation of zonal flow

The most suitable case to analyze the zonal flow growth rate is $k_y = 0$. Therefore for this case the zonal flow growth rate (79) takes the form

$$\Gamma^2 = \frac{q_y^2 I_{\mathbf{k}} k_x}{\left[(2\omega + k_x c_B)k_x + \beta\right]^2}$$
$$\times\left[(\omega + k_x c_B)^2 k_x^3 + k_x^3 k_z^2 \mathrm{v}_A^2 - (\omega k_x + \beta)^2 k_x - \frac{(\omega k_x + \beta)^3}{(\omega + k_x c_B)}\right]. \tag{80}$$

By using the solution (18), Eq. (80) becomes

$$\Gamma^2 = \frac{2q_y^2 I_{\mathbf{k}}\, k_x^2 (k_x^2 |c_B| + \beta)}{k_x^2|c_B| + \beta \mp \sqrt{(k_x^2|c_B| + \beta)^2 + 4k_x^4 c_B^2 \alpha}}, \tag{81}$$

Here, $\alpha = k_z^2 \mathrm{v}_A^2 / k_x^2 c_B^2$, the upper (minus) sign before the radical belongs to Khantadze branch ω_1, and the lower one to Rossby branch ω_2.

It is seen from Eq. (81) that Khantadze waves give no contribution ($\Gamma^2 < 0$) to the generation of zonal flow, but the maximum growth rate is achieved by Rossby waves having the dispersion $\omega = -\beta / k_x$ at $\alpha = 0$. In this case

$$\Gamma^2 = q_y^2 k_x^2 I_{\mathbf{k}}. \tag{82}$$

This value coincides with the maximum value of growth rate achieved in the problem [54, 55].

If we introduce the dimensionless variables x and y used for Eq. (27), we rewrite Eq. (81) as follows

$$\gamma = \frac{\Gamma^2}{K} = \frac{2x^2(x^2+1)}{x^2 + 1 \mp \sqrt{(x^2+1)^2 + 4x^4\alpha}}, \tag{83}$$

where the normalization constant $K = q_y^2 I_{\mathbf{k}} \beta / |c_B|$. In Fig. 2, the dependence of the function γ on wave number x for the different values of α is shown. *A* and *B* curves correspond to "–" and "+" signs before the radical in Eq. (83), respectively.

6. Magnetic field generation

From Eq. (69) it follows that

$$\frac{\overline{A}_0}{\overline{\psi}_0} = \frac{I_{\|}^{\psi}}{1 - I_{\|}^{A}}, \tag{84}$$

or taking into account that $I_{\|}^{A} \sim O(q_y^2)$, we get

$$\frac{\overline{A}_0}{\overline{\psi}_0} = I_{\|}^{\psi} . \tag{85}$$

Thus the value of the generated mean magnetic field is the order of q_y^2 in comparison with the mean zonal flow value. Using Eqs. (78) and (79) we get at $\Omega = q_y V_g$ (use also Eq. (75) for V_g):

$$\frac{\overline{A}_0}{\overline{\psi}_0} = q_y^2 k_x k_z B_{0z} \beta \frac{M}{N}, \tag{86}$$

where

$$M = 2\omega k_\perp^2 + k_\perp^2 k_x c_B + k_x \beta - 4k_y^2 \omega - 4k_y^2 (\omega + k_x c_B) - 4k_y^2 k_x \beta \frac{\omega + k_x c_B}{2\omega k_\perp^2 + k_\perp^2 k_x c_B + k_x \beta}, \tag{87}$$

$$\begin{aligned} N = {} & 4k_x k_y^2 k_z^2 \mathrm{v}_A^2 k_\perp^2 c_B \frac{2\omega k_\perp^2 + k_\perp^2 k_x c_B + k_x \beta}{\omega + k_x c_B} + k_\perp^6 (\omega + k_x c_B)^3 \left(4\frac{k_y^2}{k_\perp^2} - 1 \right) \\ & - k_\perp^6 k_z^2 \mathrm{v}_A^2 (\omega + k_x c_B) + 8k_y^2 \omega k_\perp^4 k_z^2 \mathrm{v}_A^2 - 12 k_y^2 k_\perp^2 k_z^2 \mathrm{v}_A^2 (\omega k_\perp^2 + k_x \beta) \\ & + k_\perp^4 k_z^2 \mathrm{v}_A^2 (\omega k_\perp^2 + k_x \beta) + (\omega k_\perp^2 + k_x \beta)^3 . \end{aligned} \tag{88}$$

Thus, when $k_x k_z B_{0z} \beta \neq 0$ mean magnetic field is also generated along with the mean zonal flow generation.

As in the case of Eq. (80) we consider $k_y = 0$, then from Eq. (86) we get

$$\frac{\overline{A}_0}{\overline{\psi}_0} = q_y^2 \frac{k_z}{k_x} B_{0z} \beta \frac{P}{Q}, \tag{89}$$

where

$$\begin{aligned} P &= 2\omega k_x + k_x^2 c_B + \beta, \\ Q &= -k_x^3 (\omega + k_x c_B)^3 - k_x^3 k_z^2 \mathrm{v}_A^2 (\omega + k_x c_B) + k_x^2 k_z^2 \mathrm{v}_A^2 (\omega k_x + \beta) + (\omega k_x + \beta)^3 \\ &= -(k_x^2 c_B - \beta) \left[(2\omega + k_x c_B) k_x + \beta \right]^2 . \end{aligned} \tag{90}$$

Then

$$\frac{\overline{A}_0}{\overline{\psi}_0} = q_y^2 \frac{k_z}{k_x} \frac{B_{0z} \beta}{(\beta - k_x^2 c_B)[(2\omega + k_x c_B) k_x + \beta]}. \tag{91}$$

Substituting the solution (18), we get

$$\frac{\overline{A}_0}{\overline{\psi}_0} = \pm q_y^2 \frac{B_{0z} \beta |c_B|}{\mathrm{v}_A} \frac{\sqrt{\alpha}}{(\beta + k_x^2 |c_B|)[(|c_B| k_x^2 + \beta)^2 + 4k_x^4 \alpha c_B^2]^{1/2}}. \tag{92}$$

For the evaluation order we get

$$\frac{\overline{A}_0}{\overline{\psi}_0} \approx \pm \frac{q_y^2 B_{0z} |c_B|}{\mathrm{v}_A \beta} \sqrt{\alpha}. \tag{93}$$

In the dimensionless variables x and y used for Eq. (27), we get from Eq. (92)

$$\frac{\overline{A}_0}{\overline{\psi}_0} = \pm \frac{q_y^2 B_{0z} |c_B|}{\mathrm{v}_A \beta} \frac{\sqrt{\alpha}}{(1 + x^2)[(1 + x^2)^2 + 4x^4 \alpha]^{1/2}}. \tag{94}$$

In Fig. 3 the dependence of the function $\lambda = \frac{\bar{A}_0}{\bar{\psi}_0} \frac{v_A \beta}{q_y^2 B_{0z} |c_B|}$ on wave number x for the values α = 1; 5 is shown. A and B curves correspond to "+" and "–" signs in Eq. (94).

7. Discussions and conclusions

In this paper, the nonlinear generation of large – scale, and low – frequency zonal flows and magnetic fields by relatively small – scale ULF EM coupled Rossby – Alfvén – Khantadze (CRAK) planetary waves is investigated in the Earth's ionospheric E – layer. The importance of latitudinal non-homogeneity of both Coriolis parameter and the geomagnetic field along with the prevalent effect of Hall conductivity for CRAK is shown. In addition, accounting of the vertically directed propagation of the perturbations under the consideration leads to the z-dependence and the problem becomes essentially three-dimensional. As a result, owing to the existence of magnetic field perturbations, Alfvén waves also became incorporated in the dynamics of problem. Action of these effects leads to the coupled propagation of EM Rossby – Alfvén – Khantadze modes, which are described by the system of nonlinear Eqs. (6) and (15). Due to such coupling dispersion of both Alfvén and Khantadze waves appeared. Note that the long-lived (compared to linear wave packets) nonlinear structures can be formed under the condition when the waves dispersion is compensated by their nonlinearity.

The dispersion relation for the linear EM CRAK is obtained [see Eq. (17)] and analyzed in detail in Sec. 3. The mode is composed by two branches ω_1 and ω_2. For small values of perpendicular wave number $k_\perp$ the frequencies ω_1 and ω_2 can be described analytically by Eqs. (19) and (20) while for large values of $k_\perp$ by Eq. (21). Analytical expression for the corresponding new type of Alfvén waves is given by Eq. (22). All branches of oscillations are mutually influenced. Depending on the perpendicular wave number the appropriate behavior of phase velocities $\omega_{1,2}/k_x$ for the different values of parameter $\alpha = k_z^2 v_A^2 / k_x^2 c_B^2$ is given in Fig. 1 (Curves A belong to ω_1 and B ones to ω_2). It is clarified that in case of small $k_\perp$ the phase velocity of the branch ω_1 tends to the finite value $\omega_1/k_x = |c_B|$ and corresponds to Khantadze waves, while for the branch ω_2 it tends to the $-\infty$, which corresponds to Rossby waves. For the large values of $k_\perp$ the phase velocity of the branch ω_1 tends to the finite value $\omega_1/k_x = \frac{1}{2}|c_B|(1+\sqrt{1+4\alpha})$ which is more then $|c_B|$. Thus the existence of Alfvén waves causes the increase of the phase velocity of Khantadze waves as compared with the case $\alpha = 0$. As to the case of Rossby waves for the large values of the perpendicular wave number $k_\perp$ the phase velocity of the branch ω_2 tends to the finite value $\omega_2/k_x = \frac{1}{2}|c_B|(1-\sqrt{1+4\alpha}) < 0$. Thus in this case Alfvén waves cause the increase of the phase velocity of Rossby waves as compared with the case $\alpha = 0$. Note that in case of $k_x > 0$ the branch of Khantadze waves (ω_1) propagates along the latitude circles eastward, while the branch of Rossby waves (ω_2) along the latitude circles westward against a background of mean zonal wind.

$\alpha = 1 \quad \alpha = 5 \quad \alpha = 0$

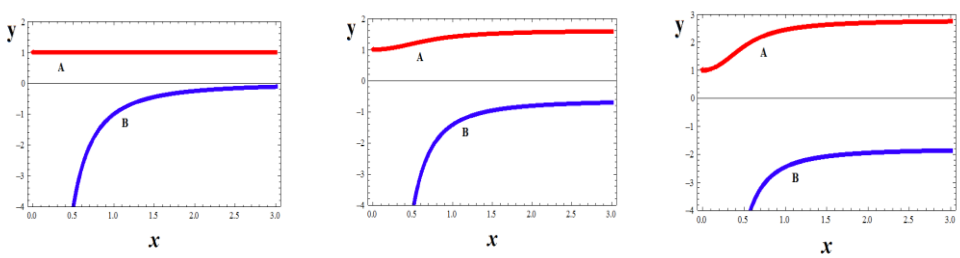

Fig. 1. Dependence of phase velocity of coupled Rossby – Alfvén – Khantadze modes on wave number x at different values of α.

Dealing with zonal flows and magnetic fields generation problem by EM CRAK modes in the weakly ionized ionospheric E-layer gas we have used the modified parametric approach [58] and the spectrum of primary modes is assumed to be arbitrary [see Eq. (31)]. Then, instead of the side-band amplitude for a single wave vector $\boldsymbol{k}$, we have dealt with a spectrum of such amplitudes [see Eq. (32)] and as a consequence the appropriate driving forces are presented as summation (or integration) over the spectrum of the primary modes [see Eq. (39) and (40)]. The developed method can be effectively used for different types of primary modes having arbitrary spectrum broadening. To describe the nonlinear dynamics of the zonal flows and magnetic fields generation by EM CRAK waves the appropriate system of coupled equations is obtained [see Eqs. (35) and (36)]. We have shown that these equations are unstable to four wave parametric instability and the coherent, monochromatic CRAK waves can drive a band of modes and corresponding zonal flow and magnetic field unstable. Thus, we have investigated the interaction of a pump CRAK modes, two their satellites (side-band waves) and a sheared zonal flow. For the monochromatic wave packet the instability [see Eq. (78)] is of the hydrodynamic type. The nonlinear instability mechanism is driven by the vorticity advection leading to the inverse energy cascade toward the longer wavelength. Consequently, short wavelength turbulence of CRAK waves is unstable causing the excitation of low-frequency and large-scale perturbations of the zonal flow and magnetic field. It is shown that in the system of Eqs. (35) and (36) controlling the evolution of zonal flow and magnetic field the driving mechanism of the instability is associated with the mixture of mean Reynolds and Maxwell stresses $R_{\perp}$ [see Eq. (37)] and mean electromotive force $R_{\parallel}$ [see Eq. (38)], respectively.

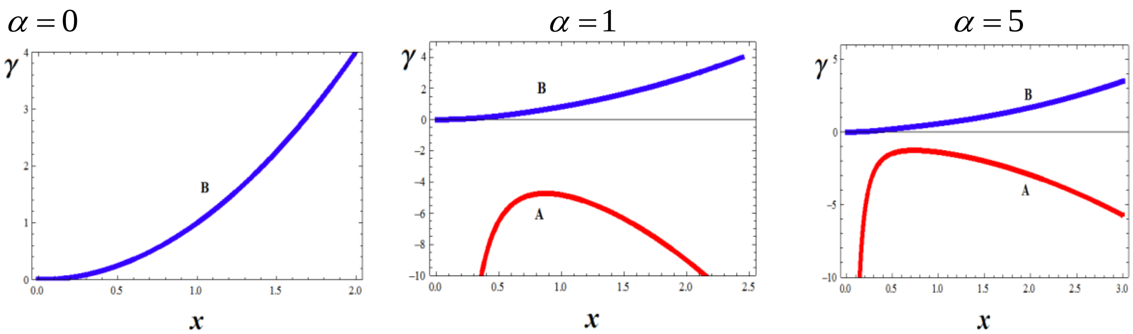

Fig. 2. Dependence of the function γ on wave number x at different values of α.

We studied the propagation of zonal flow along the geographical parallels when the corresponding mean flow velocity depends only on the meridional *y*-coordinates. From our investigations it is seen that the maximum growth rate of the zonal flow generation is achieved at $k_y = 0$, when the group velocity $V_g = 0$ [see Eq. (75)] and therefore the real part of oscillations for zonal flow becomes zero. In this case the excitation of zonal flow is stipulated only by Rossby waves and the corresponding growth rate is (see Eq. (82))

$$\Gamma \approx \left| q_y k_x r_R^3 \beta \tilde{\psi}_+ \right|, \tag{95}$$

which is equal to the maximum growth rate achieved in the problems [Kaladze et al. 2009, 2012]. In Eq. (95) the stream function $\tilde{\psi}_+$ of pump modes is normalized by $\mathrm{v}_R r_R$, where $\mathrm{v}_R = \beta r_R^2$ is the Rossby velocity and $r_R = c_s / f$ (c_s is the equivalent sound speed in the ionospheric *E*-layer) is the Rossby radius, respectively. Here for this regime, we have $q_y r_R \sim 0.1, k_x r_R \sim 10, r_R \approx 10^6 m, \beta \approx 10^{-11} m^{-1} s^{-1}$, and $\tilde{\psi}_+ \sim 10^{-2}$. Then, the numerical value for the zonal flow growth rate becomes $\Gamma \approx 10^{-7} s^{-1}$. This estimation is consistent with existing observations, and conducted investigations provide the essential nonlinear mechanism for the driving spectral energy from short-scale CRAK waves to large-scale reinforced zonal flows in the Earth's ionosphere.

In Fig.2 the dependence of the squared dimensionless growth rate γ on the wave number *x* (see Eq. (83)) for the different values of α is shown (curves A belong to the branch ω_1 and *B* ones to ω_2). It is seen that Khantadze waves (ω_1) don't contribute in the generation of zonal flow, for them $\gamma \leq 0$. The maximum growth rate is achieved for the Rossby waves branch ω_2 at $\alpha = 0$. This is the case when Alfvén waves also don't contribute in the growth rate. Thus the generation of zonal flow is mainly stipulated by Rossby waves. With increase of α the growth rate is decreasing in accordance with Eq. (83).

Here, the mean magnetic field excitation has the special attention and its dynamics is described with detail in Sec. 6. Generated magnetic field is of the order q_y^2 with respect to the excited mean zonal flow and is caused only by the existence of Alfvén waves. Excited mean magnetic field has the prevalent component b_y (as in the calculations we gave the priority to $k_y = 0$ consideration) and as the zonal flow is sheared in the meridional *y*-direction. It is found that the ratio of the mean magnetic function $\bar{A}_0$ to mean zonal flow $\bar{\psi}_0$ strongly depends on the pumping wave branches of ω (see Eq. (91)). After the substitution of ω from Eq. (18), we get Eq. (92), which shows that both Rossby (ω_2) and Khantadze (ω_1) branches give symmetric by sign contributions in the generation of the magnetic field component b_y. The following estimation for the generated magnetic field (see Eq. (93)) is valid

$$\left|\bar{b}_y\right| \approx \frac{q_y^2 B_0 \left|c_B\right|}{\mathrm{v}_A \beta r_R} \sqrt{\alpha}\,\bar{\psi}_0, \qquad (96)$$

where the Rossby radius r_R is chosen as the characteristic scale-length. Numerically, to approximate this value, we consider $\left|c_B\right| \sim (1-10)\,km/s$, $\mathrm{v}_A \sim (0.1-1)\,km/s$, $B_0 \sim 0.5\times10^{-4}T$, $\beta \approx 10^{-11} m^{-1}s^{-1}$, and consider $\bar{\psi}_0 \approx \bar{\mathrm{v}}\mathrm{r_R}$ (where $\bar{\mathrm{v}} = (1\text{-}100)\mathrm{m/s}$) is the local ionospheric mean wind's velocity). Then, the values for the excited mean magnetic field becomes $\left|\bar{b}_y\right| = (10^2 - 10^3)nT$. Consequently, the intensification of the geomagnetic field perturbed pulses takes place.

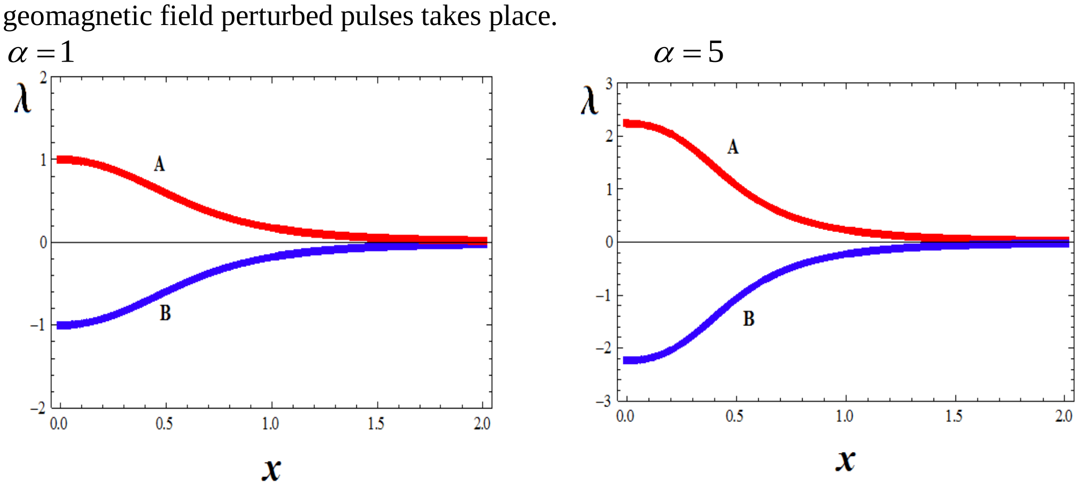

Fig. 3. Dependence of the function λ on wave number x at different values of α .

In Fig.3 the dependence of the dimensionless ratio (see Eq. (94)) on wave number *x* for the values $\alpha = 1; 5$ is shown. The curve *A* belongs to Khantadze waves contribution, while the curve *B* to Rossby waves contribution.

Note that for the large latitude in the northern hemisphere our consideration has been limited to the nearly constant dipole geomagnetic field.

Placed on the solid ground magnetometer chains register large-scale variations of exposed origin in δB. However, the incoming values are much lower than those in the *E*-layer since δB falls off exponentially below the conductive slab (e.g., [59]), i.e. $\delta B \propto \exp(-2\pi d / \lambda)$, where $d \approx 150 km$ is the characteristic scale at the *E*-layer heights. For the discussing planetary wavelengths $\lambda \approx 10^3 km$ and the estimated damping rate is of order unity. We would like to note that studied in the given paper theoretically ULF electromagnetic modes in the *E*-layer are not adequately studied experimentally and further experimental studies are required.

Thus, in this paper the conducted investigation shows that parametric instability becomes a sufficient nonlinear mechanism to drive large-scale zonal flows and intense mean magnetic field in the weakly ionized ionosphere *E*-layer.

Acknowledgments. The authors are grateful for the partial financial support from the International Space Science Institute (Bern, Switzerland) through the grant Large – scale vortices and zonal winds in planetary atmospheres/ionospheres: Theory vs. observations.

It would be worth to mention the JGGS vice-ed.'s attention to the work.

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(Received in final form 20 December 2013)

Генерирование зонального течения и магнитного поля сцеплёнными волнами Россби-Альфвена-Хантадзе в Е-слое ионосферы Земли

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Резюме

Показано, что в слабоионизированном Е-слое ионосферы Земли, где преобладает холловская проводимость плазмы, может существовать новый тип сцеплённых электромагнитных (ЭМ) планетарных волн Россби-Альфвена-Хантадзе (СРАХ), обусловленных широтной неоднородностью кориолисова параметра Земли и геомагнитного поля. Под воздействием такого сцепления возбуждается новый тип диспергирующих волн Альфвена. Исследуется генерирование сдвигового зонального течения и магнитного поля под действием СРАХ ЭМ планетарных волн. Нелинейный механизм неустойчивости основывается на параметрическом возбуждении зонального течения посредством взаимодействия четырёх волн, ведущих к инверсионному каскаду энергии в сторону более длинных волн. Выведена система 3D сцеплённых уравнений, описывающих нелинейное взаимодействие накачивающих СРАХ волн и зонального течения. Определены скорость роста соответствующей неустойчивости и условиядля их управления. Обнаружено, что рост скорости главным образом обусловлен волнами Россби, а генерация магнитного поля средней интенсивности вызывается волнами Альфвена.

დედამიწის E–ფენაში ზონალური დინების და მაგნიტური ველის გენერირება როსბი–ალფვენ–ხანთაძის გადაბმული ტალღების მეშვეობით

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ნაჩვენებია, რომ დედამიწის იონოსწფეროს სუსტად იონიზირებულ E–ფენაში, სადაც ბატონობს ქოლის გამტარობა, გენერირდება ახალი ტიპის როსბი–ალფვენ–ხანთაძის (გრახ)

გადაბმული პლანეტარული ელექტრომაგნიტური ტალღები განპირობებული დედამიწის კორიოლისის პარამეტრის და გეომაგნიტური ველის განედური არაერთგვაროვნებების არსებობით. ტალღების ამგვარი გადაბმულობის გამო აღიძვრება ახალი ტიპის ალფვენის ტალღები. შეისწავლებაზონალური დინების წანაცვლების და (გრახ ემ) პლანეტარული ტალღების წარმოშობა. არამდგრადობის არაწრფივი მექანიზმი ეყრდნობა ზონალური ნაკადის პარამეტრულ აღძვრას ოთხი ტალღის ურთიერთქმედებით, რომელსაც მივყავართ ენერგიის ინვერსიულ კასკადისკენ უფრო გრძელი ტალღების მიმართულებით. გამოყვანილია 3D გადაბმულ განტოლებათა სისტემა, რომელიც აღწერს მქაჩავი (გრახ) ტალღების არაწრფივ ურთიერთობას ზონალურ დინებასთან. განსაზღვრულია სათანადო არამდგრადობის ზრდის სიჩქარე და მისი მართვის პირობები. მიღებულია, რომ სიჩქარის ზრდა ძირითადად განისაზღვრება როსბის ტალღების მოქმედებით, ხოლო მაგნიტური ველის წარმოშობა – ალფვენის ტალღების მოქმედებით.